Ch. 12  Graphing and Optimization

12.1  First Derivative and Graphs

1  Increasing and Decreasing Functions

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Provide an appropriate response.

1) Identify the intervals where \( f(x) \) is increasing.

![Graph showing intervals where \( f(x) \) is increasing]

2) Identify the intervals where \( f'(x) < 0 \).

![Graph showing intervals where \( f'(x) < 0 \)]
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the given graph of $f(x)$ to find the intervals on which $f'(x) > 0$.

3)

A) $f(x) > 0$ on $[-3, \infty)$, $f'(x) < 0$ on decreasing on $(-\infty, -3]$  
B) $f'(x) < 0$ on $(-\infty, \infty)$  
C) $f'(x) > 0$ on $(-\infty, -7] \cup [1, \infty)$, $f'(x) < 0$ on $[-7, 1]$  
D) $f'(x) > 0$ on $[-7, \infty)$, $f'(x) < 0$ on $(-\infty, -7]$  

4)

A) $f'(x) > 0$ on $[-5, 5]$, $f'(x) < 0$ on $(-\infty, -5] \cup [5, \infty)$  
B) $f'(x) > 0$ on $(-\infty, -5] \cup [5, \infty)$, $f'(x) < 0$ on $[-5, 5]$  
C) $f'(x) > 0$ on $(-\infty, 5]$, $f'(x) < 0$ on $[5, \infty)$  
D) $f'(x) > 0$ on $[-25, 25]$, $f'(x) < 0$ on $(-\infty, -25] \cup [25, \infty)$
5) Find the critical values and determine the intervals where \( f(x) \) is increasing and the intervals where \( f(x) \) is decreasing for the function \( f(x) = x^3 + 3x^2 - 24x + 6 \).
   A) increasing on \(( -\infty, -4) \cup (2, \infty)\); decreasing on \((-4, 2)\)
   B) decreasing on \(( -\infty, -4) \cup (2, \infty)\); increasing on \((-4, 2)\)
   C) increasing on \((-\infty, -4)\); decreasing on \((-4, 2)\)
   D) increasing on \(( -\infty, -4) \cup (2, \infty)\); decreasing on \((-4, \infty)\)

6) Determine the intervals for which the function \( f(x) = x^3 + 18x^2 + 2 \), is decreasing.
   A) \((-12, 0)\)  B) \((-\infty, -12) \cup (0, \infty)\)  C) \((-\infty, -12) \cup (-12, 0)\)  D) \((0, 12) \cup (12, \infty)\)

7) Determine the interval(s) where \( f(x) = \frac{x^2}{x - 3} \) is decreasing.
   A) \((0, 3) \cup (3, 6)\)  B) \((0, 6)\)  C) \((-\infty, 0) \cup (6, \infty)\)  D) \((0, 3) \cup (6, \infty)\)

8) Find the critical values and determine the intervals where \( f(x) \) is decreasing and the intervals where \( f(x) \) is increasing for \( f(x) = 3x^4 - 6x^2 + 7 \).
   A) increasing on \((-1, 0) \cup (1, \infty)\); decreasing on \((-\infty, -1) \cup (0, 1)\)
   B) decreasing on \((-1, 0) \cup (1, \infty)\); increasing on \((-\infty, -1) \cup (0, 1)\)
   C) increasing on \((-1, 0)\); decreasing on \((-\infty, -1) \cup (0, 1)\)
   D) decreasing on \((-1, 0) \cup (1, \infty)\); increasing on \((-\infty, -1)\)

9) Use a graphing utility to approximate the intervals where \( f(x) \) is decreasing and intervals where \( f(x) \) is increasing for the function \( f(x) = x^4 - 3x^3 - 2x^2 + 5x \). Round your answer to two decimal places.
   A) decreasing on \((-\infty, -0.82) \cup (0.62, 2.45)\); increasing on \((-0.82, 0.62) \cup (2.45, \infty)\)
   B) increasing on \((-\infty, -0.82) \cup (0.62, 2.45)\); decreasing on \((-0.82, 0.62) \cup (2.45, \infty)\)
   C) decreasing on \((-\infty, -0.82)\); increasing on \((-0.82, 0.62)\)
   D) increasing on \((-\infty, -0.82)\); decreasing on \((-0.82, 0.62)\)

10) Find the critical values and determine the intervals where \( f(x) \) is increasing and \( f(x) \) is decreasing if \( f(x) = 1 + \frac{3}{x} + \frac{2}{x^2} \).
    A) increasing on \((-\frac{4}{3}, 0)\); decreasing on \((-\infty, -\frac{4}{3}) \cup (0, \infty)\)
    B) decreasing on \((-\frac{4}{3}, 0)\); increasing on \((-\infty, -\frac{4}{3}) \cup (0, \infty)\)
    C) increasing on \((-4, 0)\); decreasing on \((-\infty, -4) \cup (0, \infty)\)
    D) decreasing on \((-4, 0)\); increasing on \((-\infty, -4) \cup (0, \infty)\)
11) Find the critical values and determine the intervals where \( f(x) \) is decreasing for \( f(x) = 3(x - 4)^{2/3} + 6 \).

A) \( f(x) \) is decreasing on \((-\infty, 4)\); increasing on \((4, \infty)\)

B) \( f(x) \) is increasing on \((-\infty, 4)\); decreasing on \((4, \infty)\)

C) \( f(x) \) is decreasing on \((-\infty, 6)\); increasing on \((6, \infty)\)

D) \( f(x) \) is decreasing on \((-\infty, -4)\); increasing on \((-4, \infty)\)

12) The percent of concentration of a certain drug in the bloodstream \( x \) hr after the drug is administered is given by \( K(x) = \frac{4x}{x^2 + 36} \). How long after the drug has been administered is the concentration a maximum? Round answer to the nearest tenth, if necessary.

A) 6 hr  B) 3.6 hr  C) 4 hr  D) 1.8 hr

2 Local Extrema

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Provide an appropriate response.

1) Use the first derivative test to determine the local extrema, if any, for the function: \( f(x) = 3x^4 - 6x^2 + 7 \).

A) local max at \( x = 0 \) and local min at \( x = -1 \) and \( x = 1 \)

B) local min at \( x = 0 \) and local max at \( x = -1 \) and \( x = 1 \)

C) local max at \( x = 1 \) and local min at \( x = 0 \)

D) local max at \( x = -1 \) and local min at \( x = 0 \) and \( x = 1 \)

2) Given \( f(x) = x + \frac{16}{x}, x < 0 \), find the values of \( x \) corresponding to local maxima and local minima.

A) local maximum at \( x = -4 \) (no local minimum)

B) local minimum at \( x = -4 \) (no local maximum)

C) local maximum at \( x = -4 \), local minimum at \( x = 4 \)

D) no local maximum or minimum

3) Use a graphing utility to approximate where the local extrema of the function \( f(x) = x^4 - 3x^3 - 2x^2 + 5x \) are to two decimal places.

A) local max at \( x \approx 0.62 \); local min at \( x \approx -0.82 \) and \( x \approx 2.45 \)

B) local min at \( x \approx 0.62 \); local max at \( x \approx -0.82 \) and \( x \approx 2.45 \)

C) local max at \( x \approx 0.82 \)

D) local min at \( x \approx -0.62 \) and \( x \approx 2.45 \)

4) Use the first derivative test to determine the local extrema, if any, for the function: \( f(x) = 3(x - 4)^{2/3} + 6 \).

A) \( f(x) \) has a local minimum at \( x = 4 \).

B) \( f(x) \) has a local maximum at \( x = 4 \).

C) \( f(x) \) has a local minimum at 6

D) \( f(x) \) has no local extrema
3 First Derivative Test

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Provide an appropriate response.

1) The critical values of \( f(x) = 4x^3 - 48x + 24 \) are \( x = -2 \) and \( x = 2 \). Use the first derivative test to determine which of the critical values correspond to a local minimum.
   A) \( x = 2 \)
   B) \( x = -2 \)
   C) \( x = 2 \) and \( x = -2 \)
   D) neither \( x = 2 \) nor \( x = -2 \) correspond to a local minimum

2) Use the first derivative test to determine the local extrema, if any, for the function: \( f(x) = x^3 + 3x^2 - 24x + 6 \)
   A) local max at \( x = -4 \) and local min at \( x = 2 \)
   B) local max at \( x = -4 \)
   C) local min at \( x = 2 \)
   D) local max at \( x = 2 \) and local min at \( -4 \)

3) The critical values of \( f(x) = 4x^3 - 48x + 24 \) are \( x = -2 \) and \( x = 2 \). Use the first derivative test to determine which of the critical values correspond to a local maximum.
   A) \( x = -2 \)
   B) \( x = 2 \)
   C) \( x = 2 \) and \( x = -2 \)
   D) \( x = 0 \) and \( x = 2 \)

Sketch a graph of the function.

4) \( f(x) = 4x^2 + 24x \)

A) \( x = 10 \)
B) \( x = 2 \)
C) \( x = 2 \) and \( x = -2 \)
D) \( x = 0 \) and \( x = 2 \)
5) \( f(x) = 2x^3 + 15x^2 + 24x \)
6) \( f(x) = 12x - x^3 \)
7) \( f(x) = 3x^4 - 12x^3 \)
8) \( f(x) = x^4 - 2x^2 + 3 \)

9) With \( x \) representing the water temperature in degrees Celsius, \( S(x) = -x^3 - 9x^2 + 165x + 1300, \ 5 \leq x \leq 20 \) is an approximation to the number of salmon swimming upstream to spawn. Find the temperature that produces the maximum number of salmon.

A) 5°C  B) 20°C  C) 6°C  D) 19°C

10) The Olympic flame at the 1992 Summer Olympics was lit by a flaming arrow. As the arrow moved \( d \) feet horizontally from the archer, assume that its height \( h(d) \), in feet, was approximated by the function

\[ h(d) = -0.002d^2 + 0.7d + 6.9. \]

Find the relative maximum of the function.

A) (175, 68.15)  B) (350, 129.4)  C) (0, 6.9)  D) (175, 61.25)
4 Applications to Economics

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) The annual revenue and cost functions for a manufacturer of grandfather clocks are approximately
   \( R(x) = 480x - 0.02x^2 \) and \( C(x) = 160x + 100,000 \), where \( x \) denotes the number of clocks made. What is the
   maximum annual profit?
   A) $1,180,000   B) $1,280,000   C) $1,380,000   D) $1,480,000

2) The cost of manufacturing \( x \) electric woks in one day is given by \( C(x) = 2x^3 - 16x^2 + 4x \). Find the average cost
   per electric wok and the interval where the average cost per electric wok is decreasing.
   A) \( C(x) = 2x^2 - 16x + 4 \); \( 0 < x < 4 \)
   B) \( C(x) = 6x^2 - 32x + 4 \); \( 0 < x < 4 \)
   C) \( C(x) = 6x^2 - 32x + 4 \); \( x < 4 \)
   D) \( C(x) = 2x^2 - 32x + 4 \); \( 0 < x < 4 \)

12.2 Second Derivative and Graphs

1 Using Concavity as a Graphing Tool

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Provide an appropriate response.

1) Find \( f''(x) \) for \( f(x) = -7x^9 + 5x^2 \).
   A) \( f''(x) = -504x^7 + 10 \)
   B) \( f''(x) = -63x^8 + 10x \)
   C) \( f''(x) = 504x^8 + 10 \)
   D) \( f''(x) = 504x^7 - 10 \)

2) Find \( f''(x) \) for \( f(x) = 4x - 6 \).
   A) \( f''(x) = 0 \)
   B) \( f''(x) = 4x^3 - 6x^2 \)
   C) \( f''(x) = \frac{4}{x} \)
   D) \( f''(x) = 4 \)

3) Find \( f''(x) \) for \( f(x) = 5x^4 - 6x^2 + 7 \).
   A) \( f''(x) = 60x^2 - 12 \)
   B) \( f''(x) = 20x^2 - 12 \)
   C) \( f''(x) = 60x^2 - 12x \)
   D) \( f''(x) = 20x^2 - 12x \)

4) Find \( f''(x) \) for \( f(x) = (4x + 5)^3 \).
   A) \( f''(x) = 384x + 480 \)
   B) \( f''(x) = 24x + 30 \)
   C) \( f''(x) = 12x + 15 \)
   D) \( f''(x) = 4x + 5 \)

5) Find \( y'' \) for \( y = \frac{1}{3x + 4} \).
   A) \( y'' = -\frac{18}{(3x + 4)^3} \)
   B) \( y'' = -\frac{2}{(3x + 4)^3} \)
   C) \( y'' = -\frac{6}{(3x + 4)^3} \)
   D) \( y'' = \frac{18}{(3x + 4)^3} \)

6) Find \( y'' \) for \( y = 2x^{3/2} - 6x^{1/2} \).
   A) \( y'' = \frac{3}{2} x^{-1/2} + \frac{3}{2} x^{-3/2} \)
   B) \( y'' = 3x^{1/2} - 3x^{-1/2} \)
   C) \( y'' = \frac{3}{2} x^{1/2} + \frac{3}{2} x^{-1/2} \)
   D) \( y'' = 3x^{-1/2} + 3x^{-3/2} \)
7) Find \( y'' \) for \( y = \sqrt{5x^2 + 4} \).

A) \( y'' = \frac{20}{(5x^2 + 4)^{3/2}} \)
B) \( y'' = -\frac{1}{4(5x^2 + 4)^{3/2}} \)
C) \( y'' = -\frac{25x^2}{(5x^2 + 4)^{1/2}} \)
D) \( y'' = -\frac{25x^2}{(5x^2 + 4)^{3/2}} \)

8) Find \( y'' \) for \( y = x^4 - 8x^{1/2} \).

A) \( 12x^2 + \frac{2}{x\sqrt{x}} \)
B) \( 4x^3 - \frac{4}{\sqrt{x}} \)
C) \( 12x^2 - \frac{4}{\sqrt{x}} \)
D) \( 4x^3 + \frac{4}{\sqrt{x}} \)

9) Determine the interval(s) over which \( f(x) = (x - 4)^3 \) is concave downward.

A) \( (-\infty, 4) \)
B) \( (-\infty, -4) \)
C) \( (4, \infty) \)
D) \( (-4, \infty) \)

10) Determine the interval(s) over which \( f(x) = (x + 3)^3 \) is concave upward.

A) \( (-\infty, -3) \)
B) \( (-\infty, 3) \)
C) \( (-\infty, \infty) \)
D) \( (-3, \infty) \)

2 Find Inflection Points

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Provide an appropriate response.

1) Find all inflection points for \( f(x) = x^4 - 10x^3 + 24x^2 + 3x + 5 \).

A) Inflection points at \( x = 1, x = 4 \)
B) Inflection points at \( x \approx -0.06, x \approx 2.43, x \approx 5.13 \)
C) Inflection points at \( x = 0, x = 1, x = 4 \)
D) This function does not have any inflection points.

2) Find the inflection point(s) for \( f(x) = x^3 - 6x - 1 \).

A) \( (0, -1) \)
B) \( (0, -6) \)
C) \( (1, -1) \)
D) \( (-1, 6) \)

3) Find the inflection point(s) for \( f(x) = \sqrt{x + 7} \).

A) \( (-7, 0) \)
B) \( (-3, 2) \)
C) \( (-6, 1) \)
D) There are no points of inflection.

4) Find the inflection point(s) for \( f(x) = \frac{1}{4}x^4 - x^3 + 6 \).

A) \( (0, 6) \) and \( (2, 2) \)
B) \( (0, 0) \) and \( (2, 2) \)
C) \( (0, 0) \)
D) \( (0, 6) \) and \( (2, -4) \)
3 Analyzing Graphs

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Decide if the given value of \( x \) is a critical number for \( f \), and if so, decide whether the point for \( x \) on \( f \) is a relative minimum, relative maximum, or neither.

1) \( f(x) = (x + 3)^4; \ x = -3 \)
   
   A) Critical number; minimum at \((-3, 0)\)  
   B) Critical number; maximum at \((-3, 0)\)  
   C) Critical number but not an extreme point.  
   D) Not a critical number.

Sketch the graph and show all local extrema and inflection points.

2) \( f(x) = \frac{6x}{x^2 + 9} \)
   
   A) Local minimum: \((-3, -1)\)  
   Local maximum: \((3, 1)\)  
   Inflection points: \((0, 0)\), \((-3\sqrt{3}, -\frac{3}{2}\sqrt{3})\), \((3\sqrt{3}, \frac{3}{2}\sqrt{3})\)  
   
   B) Local minimum: \((-3, -\frac{1}{2})\)  
   Local maximum: \((3, \frac{1}{2})\)  
   Inflection point: \((0, 0)\)  

C) Maximum: \((0, \frac{2}{3})\)  
No inflection point  

D) Local minimum: \((3, -1)\)  
Local maximum: \((-3, 1)\)  
Inflection point: \((0, 0)\)
4 Curve Sketching

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Sketch the graph and show all local extrema and inflection points.

1) \( f(x) = \frac{1}{\sqrt{9-x^2}} \)

A) Local min: \((0, \frac{1}{3})\)
No inflection point

B) Local max: \((0, \frac{1}{3})\)
No inflection point

C) Local max: \((0, 1)\)
No inflection point

D) Local min: \((0, 1)\)
No inflection point
2) \( f(x) = 7x^2 + 14x \)

A) Min: \((-1, -7)\)
No inflection points

B) Min: \((1, -7)\)
No inflection points

C) Min: \((-2, -14)\)
No inflection points

D) Min: \((2, -14)\)
No inflection points
3) \( f(x) = 2x^3 + 12x^2 + 18x \)

A) Local max: (-3, 0), min: (-1, -8)
Inflection point: (-2, -4)

B) Local min: (2, 10)
No inflection point

C) No extrema
Inflection point: (0, 0)

D) Local maximum: (0, 0)
Local minimum: (-7, 343)
Inflection point: (-3.5, 171.5)
5 Point of Diminishing Returns

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Provide an appropriate response.

1) A backpack manufacturer is planning to expand its work force. They estimate that the number of backpacks produced by hiring new workers is given by \( T(x) = -0.25x^4 + 4x^3, \ 0 \leq x \leq 12. \) Determine when the rate of backpacks is increasing and when it is decreasing. Determine the point of diminishing returns and the maximum rate of change of backpack production.

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

2) A company estimates that it will sell \( N(t) \) hair dryers after spending \( \$t \) thousands on advertising as given by:
\[
N(t) = -3t^3 + 450t^2 - 21,600t + 1,100, \quad 40 \leq t \leq 60
\]
For which values of \( t \) is the rate of sales \( N'(t) \) increasing?
A) \( 40 < t < 50 \) \quad B) \( 40 < t < 60 \) \quad C) \( 50 < t < 60 \) \quad D) \( t > 40 \)

3) Because of material shortages, it is increasingly expensive to produce 6.2L diesel engines. In fact, the profit in millions of dollars from producing \( x \) hundred thousand engines is approximated by
\[
P(x) = -x^3 + 30x^2 + 10x - 52, \ \text{where} \ 0 \leq x \leq 20. \] Find the inflection point of this function to determine the point of diminishing returns.
A) (10, 2048) \quad B) (10, 114.67) \quad C) (10, 1958) \quad D) (7.50, 2048)

12.3 L'Hopital's Rule

1 L'Hopital's Rule/Indeterminate Form \( 0/0 \)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Provide an appropriate response.

1) Find \( \lim_{x \to 3} \frac{x^4 - 81}{x - 3} \).
A) 108 \quad B) 3 \quad C) 81 \quad D) 0

2) Find \( \lim_{x \to 2} \frac{x^2 + x - 2}{x^2 - 4} \).
A) \( \frac{3}{4} \) \quad B) 2 \quad C) 0 \quad D) \infty

3) Find horizontal asymptotes, if any, for \( f(x) = \frac{2x^2 - 2}{4x^3 - 3} \).
A) \( y = 0 \) \quad B) \( y = \frac{1}{2} \) \quad C) \( y = \frac{2}{3} \) \quad D) \( y = -\frac{2}{3} \)
2  One-Sided Limits and Limits at ∞

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the limit, if it exists.

1) \( \lim_{x \to \infty} \frac{3x - 2x^2 + 10x^3}{6 - 2x - x^3} \)
   
   A) -10  
   B) 10  
   C) \( \frac{3}{2} \)  
   D) ∞

2) \( \lim_{x \to -\infty} \frac{6x^3 + 2x^2}{6x^2 - x} \)
   
   A) -∞  
   B) 0  
   C) \( \frac{1}{3} \)  
   D) -6

3) Find: \( \lim_{x \to \infty} \frac{5x^2 + 3x - 1}{6x^2 - x + 7} \)
   
   A) \( \frac{5}{6} \)  
   B) \( -\frac{5}{6} \)  
   C) \( \frac{1}{7} \)  
   D) \( -\frac{1}{7} \)

3  L'Hôpital's Rule/Indeterminate Form ∞/∞

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Provide an appropriate response.

1) Find \( \lim_{x \to \infty} \frac{3x + 4}{4x^2 - 3} \).
   
   A) 0  
   B) \( \frac{3}{4} \)  
   C) \( \frac{4}{3} \)  
   D) \( -\frac{4}{3} \)

2) Find \( \lim_{x \to +\infty} \frac{x^2}{e^x} \).
   
   A) 0  
   B) ∞  
   C) - ∞  
   D) \( e^x \)

3) Find \( \lim_{x \to +\infty} \frac{\ln x}{x} \).
   
   A) 0  
   B) 1  
   C) ∞  
   D) - ∞

Page 239
Sketch a graph of the function.

4) \( f(x) = \frac{5x + 1}{x} \)

Provide an appropriate response.

5) Find vertical asymptotes for \( f(x) = \frac{7x - 2}{x^2 - 3x - 4} \)

A) \( x = -1, x = 4 \)  
B) \( x = 1, x = 4 \)  
C) \( x = 1, x = -4 \)  
D) \( x = -1, x = -4 \)
12.4 Curve Sketching

1 Graphing Strategy

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Graph the function and locate intervals on which the function is increasing or decreasing, open intervals on which the function is concave up or concave down, and all inflection points.

1) \( f(x) = x^4 e^x, \ -\infty < x < \infty \)

- A) \( f \) is increasing on \((-\infty, -4]\) and \([0, \infty)\) and decreasing on \([-4, 0]\). \( f \) is concave up on \((-\infty, -6)\) and \((-2, \infty)\) and concave down on \((-6, -2)\). \( f \) has inflection points at \( x = -6 \) and \( x = -2 \).

- B) \( f \) is increasing on \([0, 4]\) and decreasing on \((-\infty, 0]\) and \([4, \infty)\). \( f \) is concave up on \((-\infty, 2)\) and \((6, \infty)\) and concave down on \((2, 6)\). \( f \) has inflection points at \( x = 2 \) and \( x = 6 \).
C) \( f \) is increasing on \([-4, 0]\) and decreasing on \((-\infty, -4] \) and \([0, \infty)\). \( f \) is concave up on \((-6, -2)\) and concave down on \((-\infty, -6)\) and \((-2, \infty)\). \( f \) has inflection points at \(x = -6\) and \(x = -2\).

D) \( f \) is increasing on \((-\infty, -4]\) and \([0, \infty)\) and decreasing on \([-4, 0]\). \( f \) is concave up on \((-2, \infty)\) and concave down on \((-\infty, -2)\). \( f \) has an inflection point at \(x = -2\).

2) \( f(x) = \frac{x^2}{x^2 + 16}, \quad -\infty < x < \infty \)
A) 
\[
\begin{align*}
\text{f is increasing on } [0, \infty) \text{ and decreasing on } (-\infty, 0]. \\
\text{f is concave up on } \left( -\frac{4}{\sqrt{3}}, \frac{4}{\sqrt{3}} \right) \\
\text{and concave down on } \left( -\infty, -\frac{4}{\sqrt{3}} \right) \text{ and } \left( \frac{4}{\sqrt{3}}, \infty \right). \\
\text{f has inflection points at } x = -\frac{4}{\sqrt{3}} \text{ and } x = \frac{4}{\sqrt{3}}.
\end{align*}
\]

B) 
\[
\begin{align*}
\text{f is increasing on } (-\infty, 0] \text{ and decreasing on } [0, \infty). \\
\text{f is concave up on } \left( -\infty, -\frac{4}{\sqrt{3}} \right) \\
\text{and } \left( \frac{4}{\sqrt{3}}, \infty \right) \text{ and concave down on } \left( -\frac{4}{\sqrt{3}}, \frac{4}{\sqrt{3}} \right). \\
\text{f has inflection points at } x = -\frac{4}{\sqrt{3}} \text{ and } x = \frac{4}{\sqrt{3}}.
\end{align*}
\]

C) 
\[
\begin{align*}
\text{f is increasing on } [0, \infty) \text{ and decreasing on } (-\infty, 0]. \\
\text{f is concave up on } (-\infty, \infty). \\
\text{f has no inflection points.}
\end{align*}
\]

D) 
\[
\begin{align*}
\text{f is increasing on } (-\infty, 0] \text{ and decreasing on } [0, \infty). \\
\text{f is concave up on } \left( -\frac{4}{\sqrt{3}}, \frac{4}{\sqrt{3}} \right) \\
\text{and } \left( \frac{4}{\sqrt{3}}, \infty \right) \text{ and concave down on } \left( -\frac{4}{\sqrt{3}}, \frac{4}{\sqrt{3}} \right). \\
\text{f has inflection points at } x = -\frac{4}{\sqrt{3}} \text{ and } x = \frac{4}{\sqrt{3}}.
\end{align*}
\]
SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Provide an appropriate response.

3) Write the sign chart that corresponds to the following graph:

![Graph Image]

4) Write the sign chart that corresponds to the following graph:

![Graph Image]

5) Consider the function \( f(x) = -0.25x^4 - x^3 + 2 \). Determine the intervals where \( f(x) \) is increasing and decreasing, concave up and concave down and all local extrema. Use that information to obtain a sketch of the function.

6) Sketch the graph of \( f(x) = \frac{3x^2 + 2x + 5}{6x^2 + 2} \). Include sketch of all asymptotes.

7) Sketch the graph of \( f(x) = \frac{2x^2 + 5x - 3}{x^2 - 9} \). Include sketch of all asymptotes.

8) Sketch the graph of \( f(x) = x + \frac{3}{x^2} \). Include sketch of all asymptotes.
Sketch a graph of a single function that has these properties.

9) a) Continuous for all real numbers  
b) Differentiable everywhere except \(x = 0\)  
c) \(f'(x) < 0\) on \((-\infty, 0)\)  
d) \(f'(x) > 0\) on \((0, \infty)\)  
e) \(f''(x) < 0\) on \((-\infty, 0)\) and \((0, \infty)\)  
f) \(f(-2) = f(2) = 5\)  
g) \(y\)-intercept and \(x\)-intercept at \((0, 0)\)

2 Modeling Average Cost

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Provide an appropriate response.

1) The total cost, in dollars, of producing \(x\) cell phones is approximated by the function \(C(x) = 2000 - 30x + \frac{x^2}{5}\).

Find the minimum average cost.

A) The minimum average cost is $10 when \(x = 100\) cell phones.
B) The minimum average cost is $74 when \(x = 20\) cell phones.
C) The minimum average cost is $875 when \(x = 75\) cell phones.
D) The minimum average cost is $75 when \(x = 875\) cell phones.

The graphs of the first and second derivatives of a function \(y = f(x)\) are given. Select a possible graph of \(f\) that passes through the point \(P\). (NOTE: Vertical scales may vary from graph to graph.)

2) \(f'\)  
\(f''\)
Solve the problem.

3) Suppose that the total-cost function for a certain company to produce $x$ units of a product is given by $C(x) = 3x^2 + 45$. Graph the average cost function $A(x) = C(x)/x$.

![Graph of average cost function](image)

12.5 Absolute Maxima and Minima

1 Absolute Maxima and Minima

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Provide an appropriate response.

1) Find the absolute minimum value of $f(x) = \frac{e^x}{x^3}$ for $x > 0$. Round your answer to three decimal places.

A) 0.7439 at $x = 3$  
B) 3 at $x = 0.7439$  
C) 2.718 at $x = 1$  
D) 1 at $x = 2.718$
2) Find the absolute maximum value of \( f(x) = \frac{x^4}{e^x} \) for \( x > 0 \). Round your answer to three decimal places.

A) 4.689 at \( x = 4 \)  
B) 4.689 at \( x = 0 \)  
C) 4 at \( x = 4.689 \)  
D) 0.7439 at \( x = 4 \)

3) Find the absolute maximum and minimum values of \( f(x) = 9x^3 - 54x^2 + 81x + 13 \) on the interval \([-6, 2]\).

A) \( \text{max } f(x) = f(1) = 49 \)  
B) \( \text{max } f(x) = f(-6) = -4361 \)  
C) \( \text{max } f(x) = f(1) = 4361 \)  
D) \( \text{max } f(x) = f(1) = 4361 \)

4) Find the absolute maximum and minimum values of the function \( f(x) = \frac{4x}{x^2 + 1} \) on the interval \([-3, 0]\).

A) Absolute maximum is 0 at \( x = 0 \). Absolute minimum is -2 at \( x = -1 \).
B) Absolute minimum is 0 at \( x = 0 \). Absolute maximum is 2 at \( x = -1 \).
C) Absolute minimum is 0 at \( x = -1 \). Absolute maximum is -2 at \( x = 0 \).
D) Absolute maximum is 0 at \( x = 0 \). Absolute minimum is 2 at \( x = -1 \).

5) Find the absolute minimum value of \( f(x) = 5 + 4x + \frac{16}{x} \) for \( x > 0 \).

A) \( \text{min } f(x) = f(2) = 21 \)  
B) \( \text{min } f(x) = f(0) = 5 \)  
C) \( \text{min } f(x) = f(1) = 25 \)  
D) \( \text{min } f(x) = f(2) = 13 \)

6) Find the absolute maximum and absolute minimum values of the function \( f(x) = x^4 - 6x^2 \) on the interval \([0, 3]\).

A) Absolute maximum: \( f(3) = 27 \); absolute minimum: \( f(\sqrt{3}) = -9 \)
B) Absolute maximum: \( f(3) = -27 \); absolute minimum: \( f(\sqrt{3}) = -9 \)
C) Absolute maximum: \( f(0) = 0 \); absolute minimum: \( f(2) = -8 \)
D) This function has no absolute maximum or minimum on the given interval.

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

7) Find the absolute minimum value of \( f(x) = 4x \ln x - 7x \). Round your answer to three decimal places.

8) Find the absolute maximum value of \( f(x) = 3x - \ln |x| \). Round your answer to four decimal places.

9) Suppose \( f \) is a continuous function. Describe the graph of \( f \) at \((1, f(1))\) if \( f(1) = 0 \) and \( f'(x) < 0 \).

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

10) A drug that stimulates reproduction is introduced into a colony of bacteria. After \( t \) minutes, the number of bacteria is given approximately by:

\[ N(t) = 1,000 + 36t^2 - t^3, \quad 0 \leq t \leq 30 \]

At what value of \( t \) is the rate of growth maximum?

A) 12 min  
B) 6 min  
C) 30 min  
D) 24 min
11) The percent of concentration of an acid absorbed in a new manufacturing process after \( x \) hr after the acid has been mixed is given by \( A(x) = \frac{4x}{x^2 + 49} \). How long after the acid has been added is the concentration a maximum? Round answer to the nearest tenth, if necessary.

A) 7 hr  
B) 4.9 hr  
C) 4 hr  
D) 2.5 hr

2 Second Derivative and Extrema

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Provide an appropriate response.

1) Find the absolute minimum value of \( f(x) = x + \frac{9}{x} \) on \((0, \infty)\).

A) Absolute minimum is 6 at \( x = 3 \).  
B) Absolute maximum is 6 at \( x = 3 \).  
C) Absolute minimum is 3 at \( x = 6 \).  
D) Absolute maximum is 3 at \( x = 6 \).

2) Find the relative extrema of the function. List your answer(s) in terms of ordered pair(s).

\( f(x) = \frac{-8}{x^2 + 1} \)

A) Relative maximum: \((0, 8)\)  
B) Relative maximum: \((-1, 8)\)  
C) Relative minimum: \((0, -8)\)  
D) No relative extrema

3) Find the relative extrema of the function. List your answer(s) in terms of ordered pair(s).

\( f(x) = 20x^3 - 3x^5 \)

A) Relative minimum: \((-2, -64)\), Relative maximum: \((2, 64)\)  
B) Relative minimum: \((-2, -64)\), Relative maximum: \((0, 0)\)  
C) Relative maximum: \((0, 0)\), Relative minimum: \((2, 64)\)  
D) Relative minimum: \((-2, -64)\), Relative maximum: \((0, 0)\)

4) Find the absolute minimum value of \( f(x) = 4x + x^2 + 2 \) on \([0, \infty)\).

A) Absolute minimum is 2 at \( x = 0 \).  
B) Absolute minimum is 2 at \( x = 6 \).  
C) Absolute minimum is 4 at \( x = 2 \).  
D) Absolute minimum is 2 at \( x = 2 \).

5) Find the relative extrema of the function. List your answer(s) in terms of ordered pair(s).

\( f(x) = 5 - x^2 \)

A) Relative maximum: \((0, 5)\)  
B) Relative maximum: \((5, \sqrt{5})\)  
C) Relative minimum: \((0, 5)\)  
D) Relative minima: \((-\sqrt{5}, 0); (\sqrt{5}, 0)\)
12.6 Optimization

1 Area and Perimeter

MULTIPLE CHOICE.  Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) A carpenter is building a rectangular room with a fixed perimeter of 140 ft. What are the dimensions of the largest room that can be built? What is its area?

A) 35 ft by 35 ft; 1225 ft$^2$
B) 70 ft by 70 ft; 4900 ft$^2$
C) 35 ft by 105 ft; 3675 ft$^2$
D) 14 ft by 126 ft; 1764 ft$^2$

2) A private shipping company will accept a box for domestic shipment only if the sum of its length and girth (distance around) does not exceed 114 in. What dimensions will give a box with a square end the largest possible volume?

A) 19 in. × 19 in. × 38 in.
B) 38 in. × 38 in. × 38 in.
C) 19 in. × 38 in. × 38 in.
D) 19 in. × 19 in. × 95 in.

3) A company wishes to manufacture a box with a volume of 36 cubic feet that is open on top and is twice as long as it is wide. Find the width of the box that can be produced using the minimum amount of material. Round to the nearest tenth, if necessary.

A) 3.1 ft
B) 6.2 ft
C) 3.5 ft
D) 7 ft
2 Revenue and Profit

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Provide an appropriate response.

1) A 60 room hotel is filled to capacity every night at a rate of $40 per room. The management wants to determine if a rate increase would increase their profit. They are not interested in a rate decrease. Suppose management determines that for each $2 increase in the nightly rate, five fewer rooms will be rented. If each rented room costs $8 a day to service, how much should the management charge per room to maximize profit?
   A) $42
   B) $45
   C) $50
   D) The management should leave the rate as it is.

2) A company manufactures and sells x pocket calculators per week. If the weekly cost and demand equations are given by:
   \[ C(x) = 8,000 + 5x \]
   \[ p = 14 - \frac{x}{4,000}, \quad 0 \leq x \leq 25,000 \]

   Find the production level that maximizes profit.
   A) 18,000 pocket calculators per week
   B) 14,000 pocket calculators per week
   C) 8000 pocket calculators per week
   D) 2000 pocket calculators per week

3) The annual revenue and cost functions for a manufacturer of zip drives are approximately
   \[ R(x) = 520x - 0.02x^2 \]
   \[ C(x) = 160x + 100,000, \] where x denotes the number of drives made. What is the maximum annual profit?
   A) $1,520,000
   B) $1,620,000
   C) $1,720,000
   D) $1,820,000

4) The average manufacturing cost per unit (in hundreds of dollars) for producing x units of a product is given by:
   \[ \bar{C}(x) = 2x^3 - 42x^2 + 288x + 12, \quad 1 \leq x \leq 5 \]

   At what production level will the average cost per unit be maximum?
   A) 5 units
   B) 652 units
   C) 1 unit
   D) 12 units

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

5) The financial analysis department of a software design company determined that the cost of producing x palm assistants is \( C(x) = 5000 + 3x \). The department also determined the associated price-demand equation to be
   \[ p = 23 - \frac{x}{500}, \text{ where } p \text{ is price in dollars.} \]

   a) Obtain the profit function.
   b) Determine the maximum profit.
3 Inventory Control

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Provide an appropriate response.

1) A computer software company sells 20,000 copies of a certain computer game each year. It costs the company $1.00 to store each copy of the game for one year. Each time it must produce additional copies, it costs the company $625 to set up production. How many copies of the game should the company produce during each production run in order to minimize its total storage and set-up costs?

A) 5000 copies in 4 production runs
B) 4000 copies in 5 production runs
C) 10,000 copies in 2 production runs
D) 20,000 copies in 1 production run

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

2) A logo baseball cap manufacturer has a uniform annual demand of 25,000 caps. It costs $1 to store one baseball cap for 1 year and $500 to set up the plant for production of the logo baseball caps. How many times a year should the company produce the caps in order to minimize the total storage and set-up costs? (Assume that there are 250 working days per year.)
Ch. 12  Graphing and Optimization
Answer Key

12.1 First Derivative and Graphs
1  Increasing and Decreasing Functions
   1) (b, d), (g, h)
   2) (a, b), (d, g)
   3) A
   4) A
   5) A
   6) A
   7) A
   8) A
   9) A
  10) A
  11) A
  12) A

2  Local Extrema
   1) A
   2) A
   3) A
   4) A

3  First Derivative Test
   1) A
   2) A
   3) A
   4) A
   5) A
   6) A
   7) A
   8) A
   9) A
  10) A

4  Applications to Economics
   1) A
   2) A

12.2 Second Derivative and Graphs
1  Using Concavity as a Graphing Tool
   1) A
   2) A
   3) A
   4) A
   5) A
   6) A
   7) A
   8) A
   9) A
  10) A

2  Find Inflection Points
   1) A
   2) A
   3) D
   4) A
3 Analyzing Graphs
1) A
2) A

4 Curve Sketching
1) A
2) A
3) A

5 Point of Diminishing Returns
1) The rate of change of backpack production is increasing when hiring between 0 and 8 new workers and decreasing when hiring between 8 and 12 new workers. The point of diminishing returns is 8 new workers and the maximum rate of change is 1024 backpacks.
2) A
3) A

12.3 L'Hopital's Rule
1 L'Hopital's Rule/Indeterminate Form 0/0
1) A
2) A
3) A
2 One-Sided Limits and Limits at ∞
1) A
2) A
3) A
3 L'Hopital's Rule/Indeterminate Form ∞/∞
1) A
2) A
3) A
4) A
5) A

12.4 Curve Sketching
1 Graphing Strategy
1) A
2) A
3) 
\[ f(x) \]
\[
\begin{array}{c|c|c}
0 & 4 & 6 \\
\hline
\end{array}
\]
4) 
\[ f(x) \]
\[
\begin{array}{c|c|c}
0 & 2 & 5 \\
\hline
\end{array}
\]
5) \( f(x) \) is increasing on \( (-\infty, -3) \), decreasing on \( (-3, 0) \cup (0, \infty) \); concave up on \( (-2, 0) \), concave down on \( (-\infty, -2) \cup (0, \infty) \). Local max is at 8.75 at \( x = -3 \).
2 Modeling Average Cost
1) A
2) A
3) A

12.5 Absolute Maxima and Minima
1 Absolute Maxima and Minima
1) A
2) A
3) A
4) A
5) A
6) A
7) \(-8.468\) at \(e^{3/4}\)
8) 7.3890 at \(x = e^2\)
9) The graph of \(f\) at \((1, f(1))\) is a local maximum.
10) A
11) A

2 Second Derivative and Extrema
1) A
12.6 Optimization

1 Area and Perimeter
   1) A
   2) A
   3) A

2 Revenue and Profit
   1) D
   2) A
   3) A
   4) A

   5) a) \[ P(x) = \frac{x^2}{500} + 20x - 5000 \]

   b) $45,000

3 Inventory Control
   1) A
   2) The company will minimize its costs by making 5000 caps five times during the year.