Reversibility and home state. Reversibility is one of the often used properties demanded by many applications.

**Definition 8.** A Petri net \((N,M_0)\) is said to be reversible if for each marking \(M \in R(M_0)\), \(M_0\) is reachable from \(M\).

Thus, in reversible net one can always get back to the initial marking or state. In many applications, it is not necessary to get back to the initial state as long as one can get to some (home) state. Therefore, we relax the reversibility condition and define a home state.

**Definition 9.** A marking \(M'\) is said to be a home state if, for each marking \(M \in R(M_0)\), \(M'\) is reachable from \(M\).

The above three properties (boundedness, liveness and reversibility) are independent of each other. For example, a reversible net can be live or not live and bounded or not bounded. Fig. 17 shows examples of eight Petri nets for various combinations of these three properties, where \(\overline{B}\), \(\overline{L}\) and \(\overline{R}\) denote the negations of boundedness \((B)\), liveness \((L)\) and reversibility \((R)\).

**Coverability.** Coverability is closely related to 1-liveness (potential firability).

**Definition 10.** A marking \(M\) in a Petri net \((N,M_0)\) is said to be coverable if, there exists a marking \(M' \in R(M_0)\) such that \(M'(p) \geq M(p)\) for all \(p \in P\).

Let \(M\) be the minimum marking to enable transition \(t \in T\). Then \(t\) is dead if and only if \(M\) is not coverable. Thus, \(t\) is 1-live if and only \(M\) is coverable.

**Persistence.** The notion of persistence is useful in the context of parallel program schemata and speed-independent asynchronous circuits. Persistency is closely related to conflict-free nets, and a safe persistent net can be transformed into a marked graph by duplicating some transitions and places.

**Definition 11.** A Petri net is said to be persistent if, for any two enabled transitions, occurrence of one transition will not disable another.

A transition in a persistent Petri net, once it is enabled, will stay enabled until it occurs. An example of persistent Petri net is illustrated in Fig. 18.
Fig. 17. Examples of Petri nets having all possible combinations of boundedness, liveness and reversibility.
Fairness. Many different notions of fairness have been proposed in the literature on Petri nets. We will consider two basic fairness concepts: bounded fairness and unconditional (global) fairness.

**Definition 12.** Two transitions $t_1$ and $t_2$ are said to be in **bounded-fair (B-fair)** relation if and only if the maximum number of times that either one can fire while another is not firing is bounded.

**Definition 13.** A Petri net $(N,M_0)$ is said to be **B-fair net** if, every pair of transitions in the net are in a B-fair relation.

**Definition 14.** A firing sequence $\sigma$ is said to be **unconditionally (globally)-fair** if, it is finite or every transition in the net appears infinitely often in $\sigma$.

**Definition 15.** A Petri net $(N,M_0)$ is said to be **unconditionally-fair net** if, every firing sequence $\sigma$ from $M \in R(M_0)$ is unconditionally fair.

There are some relationships between these two types of fairness. For example, every B-fair net is also unconditionally-fair net and every bounded unconditionally-fair net is a B-fair net. For example, a Petri net in Fig. 19 is a B-
fair net as well as an unconditionally-fair net. The one in Fig. 20, however, is
neither a B-net nor an unconditionally-fair net, since \( t_3 \) and \( t_4 \) will not appear in
an infinite firing sequence \( \sigma = t_2 \ t_1 \ t_2 \ t_1 \cdots \).

![Fig. 19. A B-fair and an unconditionally-fair Petri net.](image)

![Fig. 20. A Pet net that is neither B-fair nor an unconditionally-fair.](image)

The unbounded net shown in Fig. 21 is unconditionally-fair net but not a
B-fair net since there is not bound on the number of times that \( t_2 \) can fire without
firing others when the number of tokens in \( t_2 \) is unbounded.
3.2 Structural properties of Petri nets

Structural properties of a Petri net depend only on its structure, and not on the initial marking and the firing policy. These properties are thus of great importance when designing manufacturing systems, since they depend only on the layout, and not on the way the system will be managed, which is not known at the design level. Most of the structural properties can be easily verified by means of algebraic techniques. The structural properties of a Petri net include liveness, boundedness, conservativeness, repetitivity, consistency and controllability properties.

Liveness and boundedness. In the previous section, we have studied liveness and boundedness as behavioral properties of Petri nets. In this section, we are going to focus on relationship between behavioural and structural liveness and boundedness properties.

Definition 16. A Petri net is said to be **structurally live** if there exists an initial marking $M_0$ such that net is live.

According to this definition, A Petri net which is live is also structurally live, but the reciprocal is false. It is important to notice that, except for some particular types of Petri nets, it is impossible to verify structural liveness.

Definition 17. A Petri net is said to be **structurally bounded** if it is bounded for any initial marking $M_0$.

Fig. 20. A Pet net that is an unconditionally-fair net but not a B-fair net.
Unlike structural liveness, structural boundedness requires that the net remains bounded for all possible initial marking. A Petri net which is structurally bounded is bounded, but the reciprocal is false.

An example of a Petri net that is bounded but not structurally bounded is illustrated in Fig. 21. This Petri net is 1-bounded in the initial marking $M_0 = (1, 0)$ since $t_2$ is the only transition that is enabled in $M_0$ and its occurrence enables $t_3$, and further occurrence of $t_3$ enables $t_2$, etc. meaning that the only token in the net will be recirculating continuously between two places. Same Petri net, on the other hand, is not bounded in $M_1 = (2, 0)$. Occurrence of $t_1$ adds new tokens in the net and there is no way to reduce them by firing other transitions.

**Conservativiness.** Conservativiness is just another property that is closely related to structural boundedness property.

**Definition 18.** A Petri net is conservative, if all transitions fire token-preservingly, i.e. all transitions add exactly as many tokens to their postplaces (output places) as they subtract from their preplaces (input places).

A conservative Petri net is structurally bounded. For example Petri nets in Fig. 19-20 are conservative nets and therefore these nets are bounded, but the one illustrated in Fig. 18 is bounded and is not conservative net.

**Repetitivity.** Repetitivity is a structural property that is closely related to structural liveness property. In repetitive Petri net any transition can be fired
indefinitely. From a manufacturing point of view, it means that the same stage can be performed as many times as it is required.

**Definition 19.** A Petri net is said to be **repetitive** if there exists an initial marking $M_0$ and a firable sequence $\sigma$ in which each transition appears an unlimited number of times.

Consider Petri net shown in Fig. 22. Since transitions $t_1$ and $t_2$ occur infinitely many time on $\sigma = t_1 t_1 t_2 t_1 t_2 \cdots$ this Petri net is repetitive. Once $t_1$ and $t_2$ occur in a row, whole net becomes dead and there is no way to fire any transition. Thus, Petri net is not live in $M_0 = (10)$. It is not difficult to show that this Petri net is not live in any marking.