Q1 [7 points] Find the number of permutations that 27x27 MIN consisting of 3 stages and made of 3x3 switches can perform.

**Answer.** A 3x3 switch can generate 3! Permutations. A stage of 9 3x3 switches generates \((3!)^9\) permutations. Consequently, 3-stage 27x27 MIN performs \((3!)^{27}\) permutations.

Q2 [7 points] Is permutation

\[ \pi = \begin{pmatrix} \ldots & 17 & \ldots & 9 & \ldots \\ \ldots & 8 & \ldots & 20 & \ldots \end{pmatrix} \]

realizable in \(\Omega_{32}\)?

**Answer.** Since \(32 = 2^5\), we need to check the condition \(L(10001,01001) + M(01000,10100) < 5\) for satisfiability. Obviously, \(L(10001,01001) = 3\), \(M(01000,10100) = 0\) and therefore

\[ L(10001,01001) + M(01000,10100) = 3 < 5. \]

This is the reason the given permutation can be performed in \(\Omega_{32}\).

Q3 [8 points] Is permutation

\[ \pi = \begin{pmatrix} 35670214 \\ 01234567 \end{pmatrix} \]

realizable in \(\Omega_8\)?

**Answer.** Let’s apply the windows method. Because of the same indices in the first column (boldfaced indices) the permutation cannot be performed in \(\Omega_8\).

\[
\begin{align*}
011000 & \quad 110 & 100 & 000 \\
101001 & 010 & 100 & 001 \\
110010 & 100 & 001 & 010 \\
111011 & \Rightarrow & 110 & \Rightarrow & 101 & \Rightarrow & 011 \\
000100 & \quad 001 & 010 & 100 \\
010101 & 101 & 010 & 101 \\
001110 & 011 & 111 & 110 \\
100111 & \quad 001 & 011 & 111
\end{align*}
\]
Q4 [1 point each] Consider a four-stage pipeline. Given the periods of the stages \( \tau_1 = 5, \tau_2 = 3, \tau_3 = 6, \tau_4 = 2 \) and the latch period \( \tau_l = 1 \)

a) Find clock period \( \tau \).
b) Find the frequency \( f \).
c) Find the number of clock periods that is necessary to perform 10 tasks.
d) Find the speed-up of the pipeline obtained on realization of 10 tasks.
e) Find the efficiency of the pipeline obtained on realization of 10 tasks.
f) Find the throughput of the pipeline obtained on realization of 10 tasks.

Answer.

a) \( \tau = \max\{\tau_1, \tau_2, \tau_3, \tau_4\} + \tau_l = \max\{5, 3, 6, 2\} + 1 = 6 + 1 = 7 \).
b) \( f = \frac{1}{\tau} = \frac{1}{7} \).
c) \( n + k - 1 = 10 + 4 - 1 = 13 \).
d) \( S_4(10) = \frac{40}{13} \approx 3 \).
e) \( E_4(10) = \frac{40}{13} \cdot 4 = \frac{160}{13} \approx 12.31 \).
f) \( \eta = \frac{10}{13} \cdot 7 = 0.77 \).

Q5 [7 points] Draw 3-stage Clos network in which 1\textsuperscript{st}, 2\textsuperscript{nd} and 3\textsuperscript{rd} stages are made of (3x4)-, (3x3)- and (4x3)-switches, respectively.

Answer.