3.2 Structural Properties of Ordinary Petri Nets

Structural properties of a Petri Net depend only on its structure, and not on the initial marking and the firing policy. These properties are thus of great importance when designing manufacturing systems, since they depend only on the layout, and not on the way the system will be managed, which is not known at the design level. Most of the structural properties can be easily verified by means of algebraic techniques. The structural properties of a Petri Net include liveness, boundedness, conservativeness, repetitivity, consistency and controllability properties.

3.2.1 Liveness and Boundeness

Liveness was studied in the previous section as a behavioral property. From a structural point of view, liveness is defined as follows.

Definition. A Petri Net is said to be structurally live if there exists an initial marking $M_0$ such that $PN = (N, M_0)$ is live.

According to this definition, a Petri Net which is live is structurally live, but the reciprocal is false. It is important to notice that, except for some particular types
of Petri Nets, it is impossible to verify structural liveness.

**Definition.** A Petri Net is said to be structurally bounded if the marked Petri Net $PN = (N, M_0)$ is bounded for any initial marking $M_0$.

![Petri Net Diagram]

Fig. 3.2.1. Bounded but not structurally bounded Petri Net.

Unlike structural liveness, structural boundedness requires that the system remains bounded whatever the initial marking. A Petri Net which is structurally bounded is bounded, but the reciprocal is false. A counter example is given by the Petri Net of Fig. 6. This Petri Net is bounded if the initial marking is $M_0 = (1,0)$, but it is not bounded if the initial marking is $M_0 = (2,0)$. Thus, this Petri Net is not structurally bounded, and the related manufacturing system includes design errors.

3.2.2 Conservativeness

Another property closely related to boundedness is conservativeness.
Definition. A Petri Net is said to be conservative if there exists a column $X$ which assign to each place $p$ a positive integer weight $x(p)$ such that:

$$X' \cdot M = X' \cdot M_0, \forall M_0 \text{ and } M \in R(M_0)$$

where $X = [x(p_1), x(p_2), ..., x(p_q)]^T$. Let us consider the Petri Net of Fig. 7, and assume that the initial marking is $M_0 = [0, 0, 0, 0, 3]$. Firing sequence $< t_1, t_1, t_2, t_3 >$ leads to marking $M = [0, 0, 1, 0, 0]$ and then the firing $< t_5, t_6, t_2, t_6 >$ transforms $M$ back into $M_0$. Indeed, the Petri Net is conservative with $X = [1, 1, 3, 1, 1]^T$, which implies that one token in place $p_3$ is equivalent to three tokens in the other places.

Fig. 3.2. Conservative Petri Net.
3.2.3 Repetitivity

Repetitivity is a structural property that characterizes structural liveness. Repetitivity means that any transition of Petri Net can be fired indefinitely. From a manufacturing point of view, it means that the same operation can be performed as many times as required: this reflects a well-designed manufacturing system.

Definition. A Petri Net is said to be repetitive if there exists an initial marking $M_0$ and a firable sequence $\sigma$ in which each transition appears an unlimited number of times.

According to this definition, a Petri Net which is structurally live is repetitive. However, the reciprocal is not true. Thus, repetitivity is a necessary condition for structural liveness and, consequently, a necessary condition for liveness.

Let us consider the Petri Net of Fig. 8. Sequence $< t_1, t_1, t_2 >$ is firable and leads to many times as we want. This
implies that the Petri Net is repetitive, but it is not structurally live since sequence $< t_1, t_2 >$ can be fired $M_0(p_1)$ times, which leads to marking $M = (0, M_0(p_1))$, which is a deadlock.

3.2.4 Consistency

Consistency is a structural property that is closely linked with reversibility. It guarantees that there exists a sequence of transition which, when fired, leads to the initial marking if the Petri Net is reversible.

**Definition.** A Petri Net is said to be consistent if there exists an initial marking $M_0$, and a sequence $\sigma$ of firable transitions which contains at least once each transition and which, when fired, leads again to $M_0$. In other words:

$$ M_0 \xrightarrow{\sigma} M_0 \quad \text{and} \quad V_\sigma > 0 $$

where $V_\sigma$ is the counting vector. According to this definition, a Petri Net, which is live and reversible, is consistent, but the reciprocal does not hold. Let us consider the Petri Net shown in Fig. 6. From the initial marking $M_0$ we can obtain the fireable sequence of transitions $\sigma = < t_2, t_1, t_3 >$ which leads the Petri Net again to the initial marking $M_0$.

3.2.5 Controllability

**Definition.** A Petri Net is said to be controllable if any marking $M'$ is reachable from any initial marking $M_0$. 