We usually distinguish between the *behavioral* and the *structural* properties of Petri Nets. The former depends on the structure of the Petri Net as well as on the initial marking. Properties of a Petri Net that are dependent on the initial marking are referred to as behavioral properties. Those that are independent of the initial marking and dependent only on the structure of the Petri Net are called structural properties.

### 3.1 Behavioral properties of Petri Nets

The reachability, boundedness, security, limitation, liveness, deadlock, reversibility and safety are commonly referred to as behavioral properties.

#### 3.1.1 Reachability

When studying the dynamics of Petri Nets, it is often desirable to know if the given marking $M_1$ can be reached from the initial marking $M_0$ or if an undesirable marking $M_2$ can be avoided. Therefore, it is necessary to know if there is a sequence of occurrences that can bring the net to a new marking $M_r$. This kind of problem is referred to as a reachability problem.
**Definition.** A step $S$ is a non-empty and finite multi-set over the set of transitions $T$.

**Definition.** A finite occurrence sequence $\sigma$ is a sequence of markings and steps:

$$\sigma = M_0[S_1 > M_2[S_2 > M_3...M_n[S_n > M_{n+1}$$

such that $n \in \mathbb{N}$, and $M_i[S_i > M_{i+1}$ for all $i \in \{1, ..., n\}$, where $\mathbb{N}$ is the set of natural numbers.

**Definition.** The marking $M_r$ is said to be reachable from $M_0$ if there exists a firing sequence of transitions that will yield $M_r$, i.e.:

$$\sigma = M_0[S_0 > M_1[S_1 > M_2[...M_{r-1}[S_{r-1} > M_r,$$

which can be succinctly denoted by $M_0 \xrightarrow{\sigma} M_r$.

We then also say that that $M_r$ is reachable from $M_0$ in $r$ steps. Analogously, an infinite occurrence sequence is a sequence of markings and steps:

$$M_1[S_1 > M_2[S_2 > M_3...$$

such that $M_i[S_i > M_{i+1}$ for all $i \in \mathbb{N}$, where $\mathbb{N} = \{1, 2, ...\}$.

For the Petri that was described in the Fig.3.1, the marking $M_1 = (1,1,1,0,0)$ is immediately reachable from the initial marking $M_0 = (0,1,0,1,0)$ by the single firing of transition $t_4$. The marking $M_2 = (0,0,1,0,1)$ is reachable from the initial marking $M_0$ by the occurrence of the step $S = \{t_4, t_5\}$. 
3.1.2 Boundedness and safety

It is often desirable to know whether Petri Net model is bounded. The boundedness property is especially useful when we want to define the size of the system or to reveal some design errors.

Definition. A Petri Net is \( k \)-bounded with respect to an initial marking \( M_0 \) if the number of tokens in any of its places never exceeds \( k \) for any marking in the reachability set \( R(M_0) \), i.e. \( M(p) \leq k \), \( \forall p \in P \) and \( \forall M \in R(M_0) \).

Definition. A Petri Net is said to be bounded (or limited) if it is \( k \)-bounded for some integer \( k > 0 \).
A special case of boundedness is safety. A Petri Net is safe if it is $k$-bounded and $k=1$ (or 1-bounded). The concept of safe and bounded nets is independent of the firing sequences. Also, note that boundedness indicates a finite reachability space. Contrarily to reachability, boundedness can be easily checked by one of the analysis techniques: coverability tree which will be presented later in this course.

For instance, a Petri Net in Fig.3.2 is not bounded since $p_2$ and $p_3$ may get infinitely many tokens. Meanwhile a Petri Net in Fig.3.3, that is slightly modification of the one illustrated in Fig.3.2, is bounded. Moreover, this Petri Net is safe.
Consider Petri Net shown in Fig.3.4. In this Petri Net the number of tokens in \( p_1 \) never exceeds 46. Consequently, \( p_1 \) is 46-bounded. On the other hand, \( p_2 \) may receive at most 30 tokens. It happens only when \( t_1 \) occurs 23 times. It can be easily seen that \( t_1 \) cannot occur more than 23 times. Hence, \( p_2 \) is 30-bounded. One can verify that \( p_3 \) is 59-bounded and \( p_4 \) is 52-bounded. From the above analysis we conclude that the Petri Net in Fig.3.4 is bounded.
3.1.3 Liveness and Deadlock

**Definition.** A Petri Net is said to be live if all its transitions can be fired from any marking $M'$ reachable from the initial marking $M_0$, i.e. $\forall M \in R(M_0), \exists M' \in R(M)$ such that $\forall t \in T$ is enabled for $M'$.

**Definition.** A marking $M'$ reachable from the initial marking $M_0$ is a deadlock if none of the transitions of the Petri Net is enable. Given the state of the system, there is no sequence of transition firings in the net that will result in the enablement of the deadlocked transitions. Note that according to the definition of liveness, if a transition is not deadlocked then it must be live, which means that there exists a firing sequence such that the transition will be enabled. This implies that liveness guarantees the absence of deadlocks. In other words, every transition of the net can fire an infinite number of times. Specifically, we say a transition $t$ is live at

**Level 0:** if it can never fire. It is a dead transition.

**Level 1:** if it can fire at most once, which means that there exists a marking $M \in R(M_0)$ such that the transition is enable.

**Level 2:** if it can fire at most $n$ times, that is, for every finite positive integer $n$ there exists a firing sequence that contains $t$ at least $n$ times.
Level 3: if there exists an infinite-length firing sequence in which \(t\) occurs infinitely often but it can be blocked.

Level 4: if it can fire infinitely many times and it cannot be blocked.

A Petri Net is live at level \(i\) if every transition is live at level \(i\). The liveness that is often interest here is the one at level 4, the strongest. A Petri Net is live if all the transitions are live.

It should be noted that a transition that is live at level 4 is live at levels 3, 2, and 1. Hence, level 4 liveness implies level 3 liveness, which implies level 2 liveness, which implies level 1 liveness. Consider the net shown in Fig. 1. The levels of liveness of transitions \(t_1\), \(t_3\) and \(t_5\) are 3, 1, and 2, respectively. Notice that bounded and live Petri Net is usually called well-formed Petri Net.

3.1.4 Reversibility

Definition. A Petri Net is reversible if it is possible to come back to the initial marking whatever the marking reached from the initial marking. In other words, a Petri Net with initial state \(M_0\) is reversible if \(M_0 \in R(M) \forall M \in R(M_0)\). Therefore, the initial marking is reachable from all marking.
3.1.5 Security

Definition A Petri Net is secure if the total number of tokens in its places does not change from setting to setting. In such a Petri Net every transition has equal number incoming and outgoing arcs, that is, for all $j \mid I(t_j) = |O(t_j)|$.

The properties of liveness, boundedness, reversibility and security are independent. A net can be {live, bounded, reversible, secure}, {live, bounded, reversible, not secure}, ..., {not live, not bounded, not reversible, not secure}, resulting in 16 possible combinations.