Q1) a) Let $S$ be a set of polynomials in $P(F)$, such that no two have the same degree. Is $S$ linearly independent? Justify your answer.

b) Find two spanning sets for the set of all $3 \times 3$ symmetric matrices.

Q2) A square matrix is called skew-symmetric if $A^T = -A$. Let $W$ be the set of all $n \times n$ skew-symmetric matrices. Show that $W$ is a subspace of $M_{nn}(F)$. Find a basis for $W$. What is the dimension of $W$?

Q3) Prove the following corollary of the replacement theorem:

Let $V$ be a vector space with dimension $n$. Any finite spanning set for $V$ contains at least $n$ vectors, and a spanning set for $V$ that contains exactly $n$ vectors is a basis for $V$. (Hint: Start the proof by showing that finite spanning sets can always be reduced down to a basis.)