E.M.U. - FACULTY OF ARTS AND SCIENCES
DEPARTMENT OF MATHEMATICS

Math124 -- GEOMETRY-- Quiz 1
22nd March 2010

duration: 40 mins.

Q1) Construct, by using the compass and the straightedge only, the perpendicular bisector for a given line-segment, and explain the way you did the construction.

\[ \text{Open compass to length of AB.} \]
\[ \text{Draw 2 arcs on both sides of line segment.} \]
\[ \Delta AEC \cong \Delta BCE \text{ by SSS so } \Delta ACD = \Delta BCD. \]
\[ \Delta ACD \cong \Delta BCD \text{ by SAS so} \]
\[ |AD| = |DB| \text{ & } \hat{A}CD = \hat{C}DB = 90^\circ. \]

Q2) Show, by using Ceva’s theorem, that the altitudes (extended if needed) for an obtuse angled triangle are concurrent.

By Ceva,
\[ \frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} \text{ should be 1.} \]
\[ \frac{\cos A}{\cos B} - \frac{\cos B}{\cos C} - \frac{\cos C}{\cos A} = 1. \]
\[ \frac{-\cos B = \cos (180 - B)}{\cos B} \text{ So altitudes must be concurrent.} \]

Q3) Let \( \Delta ABC \) be a right angled triangle with \( \angle C = 90^\circ \) and with inradius \( r \). Show that

\[ r = \frac{ab}{a+b+c} = \frac{a+b-c}{2}. \]

\[ (ABC) = \frac{ar}{2} + \frac{br}{2} + \frac{cr}{2} = \frac{s}{2} (2s) = rs \]
\[ = \frac{ab}{2}. \]
\[ \frac{ar}{2} + \frac{br}{2} + \frac{cr}{2} = \frac{ab}{2}. \]
\[ \therefore \ r = \frac{ab}{a+b+c}. \]
\[ \frac{ab}{a+b+c} = \frac{a+b-c}{2} \Longleftrightarrow 2ab = a^2 + ab - ac + b^2 - bc + ac + bc - c^2 \]
\[ a^2 + b^2 = c^2. \]