Q1) Determine for what values of \( a \in \mathbb{R} \), the linear system

\[
\begin{align*}
  x + y + az &= 1 \\
  x + ay + z &= 1 \\
  ax + y + z &= 1
\end{align*}
\]

has

a) no solution
b) unique solution
c) infinitely many solutions.
Q2) Let \( A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 3 & 8 & 7 \end{bmatrix} \). Express the matrix \( A^{-1} \) as a product of elementary matrices.
Q3) Consider the following linear system:

\[
\begin{align*}
    x & - 3y + z = 4 \\
    2x & - y = -2 \\
    4x & - 3z = 0
\end{align*}
\]

a) Find the inverse of the coefficient matrix \( A \).

b) Solve the system, by using the inverse of \( A \).
Q4) Decide whether the given matrix is invertible. If so, use Adjoint method to find its inverse.

\[
A = \begin{bmatrix}
2 & 0 & 3 \\
0 & 3 & 2 \\
-2 & 0 & -4
\end{bmatrix}
\]
Q5)

a) Find the following determinant, by reducing the matrix to **Row-Echelon Form**:

\[
\begin{pmatrix}
2 & 1 & 3 & 1 \\
1 & 0 & 1 & 1 \\
0 & 2 & 1 & 0 \\
0 & 1 & 2 & 3
\end{pmatrix}
\]

b) By using the properties of determinants, show that

\[
\begin{vmatrix}
b + c & c + a & b + a \\
a & b & c \\
2 & 2 & 2
\end{vmatrix} = 0
\]
Q6)

a) Prove that, if $A$ is $n \times n$ matrix, then

$$\det(\text{adj}(A)) = (\det(A))^{n-1}$$

b) Show that, if $B$ is a square matrix, then

i. $BB^T$ is symmetric.

ii. $B + B^T$ is symmetric.