1. A box contains a fair coin; $C_1$, and a two–headed coin; $C_2$. First a coin is selected at random from the box and tossed. If head appears, then the other coin is tossed. If tail appears, then the same coin is tossed. 
   a) Find the probability that head appears on the second toss. 
   b) If head appears on the second toss, what is the probability that it also appeared on the first toss. 

2. Given two boxes as follows:
   - Box A contains 5 red and 3 white balls.
   - Box B contains 2 red and 4 white balls.

First a fair die is tossed. If 5 or 6 appears, two balls are drawn from Box B, without replacement. Otherwise, two balls are drawn at random from Box A, without replacement.
   a) What is the probability that both balls are red? 
   b) What is the probability that the selected both balls are of the same color? 

3. An urn contains 6 white and 4 black marbles. A marble is selected at random from the urn, discarded, and two marbles of other color are put into the urn. Then a second marble is selected from the urn. Find the probability that 
   a) the second marble is black. 
   b) both marbles are black. 

4. Given two boxes as follows:
   - Box A contains 4 red and 2 white balls. 
   - Box B contains 3 red and 3 white balls. 

First a box is selected, at random. If Box B is selected, two balls are selected at random, without replacement. If Box A is selected, two balls are selected at random, with replacement. 
   a) What is the probability that both balls are of different color? 
   b) If the selected both balls are of the different color, what is the probability that they came from Box B? 
   c) What is the probability that the selected both balls are white? 

5. Given two boxes as follows:
   - Box A contains 4 red and 2 white balls. 
   - Box B contains 1 red and 3 white balls. 

First a fair die is tossed. If 1 or 6 appears, a ball is selected from Box B, put into Box A, and then a ball is selected from Box A. Otherwise, a ball is selected at random from Box A, put into Box B, and then a ball is selected from Box B. 
   a) What is the probability that both balls are white? 
   b) If the selected both balls are white, what is the probability that they came from Box A? 

6. Given two boxes as follows: 
   - Box A contains nine cards numbered 1 to 9. 
   - Box B contains five cards numbered 1 to 5. 

A box is selected at random, and a card is drawn. If the card shows an even number, another card is drawn from the same box. If the card shows an odd number, a card is drawn from other box.
   a) What is the probability that both cards show even numbers? 
   b) If both cards show even numbers, what is the probability that they came from Box A? 
   c) What is the probability that both cards show odd numbers? 

7. Three machines; A, B and C, produce 60%, 30% and 10%, of the total number of products of a factory, respectively. The defective output of these machines are 4%, 3% and 2%, respectively. A product is selected at random and found defective. What is the probability that the product was produced by machine B? 

8. In a certain college 25% of the students failed mathematics, 15% of the students failed chemistry, and 10% of the students failed both mathematics and chemistry. A student is selected at random.
   a) If the failed mathematics, what is the probability that he failed chemistry? 
   b) If the failed chemistry, what is the probability that he failed mathematics? 
   c) What is the probability that he failed mathematics and chemistry? 

9. In a certain town, 40% of the people have brown hair, 25% have brown eyes and 15% have both brown hair and brown eyes. A person is selected at random from the town.
   a) If he has brown hair, what is the probability that he also has brown eyes? 
   b) If he has brown eyes, what is the probability that he also has brown hair? 
   c) What is the probability that he he has neither brown hair nor brown eyes? 

10. In a school, 4% of boys and 1% of the girls are taller than 1.80 m. Furthermore, 60% of the students are girls. If a student is selected at random and is taller than 1.80 m., what is the probability that the student is a boy? 

11. A man is dealt 5 cards from a pack of 52 playing cards. If all 5 cards are diamond, what is the probability that they are less than 7?
12. Urn A contains 5 red and 3 white marbles, and urn B contains 2 red and 6 white marbles.
   a) If a marble is selected from each urn, what is the probability that they are of the same color?
   b) If two marbles are drawn from each urn, what is the probability that all four marbles are of the same color?

13. A coin is tossed 3 times. Let $X$ be a random variable defined by ‘number of heads’.
    a) Construct the probability distribution table for $X$.
    b) Find the expected number of heads.
    c) Sketch the probability distribution of $X$.

14. A coin is tossed 3 times. Let $Y$ be a random variable defined by ‘number of heads’ – ‘number of tails’.
    a) Construct the probability distribution table for $Y$.
    b) Compute the expectation of $Y$.
    c) Sketch the probability distribution of $Y$.

15. A pair of dice is tossed. Let $X(a,b) = \max(a,b)$.
    a) Construct the probability distribution table for $X$.
    b) Compute the expectation of $X$.
    c) Sketch the probability distribution of $X$.

16. A pair of dice is tossed. Let $Y$ be a random variable defined by $Y(a,b) = a + b$.
    a) Construct the probability distribution table for $Y$.
    b) Compute the expectation of $Y$.
    c) Sketch the probability distribution of $Y$.

17. A box contains 3 red and 9 white balls. 3 balls are selected at random without replacement, and number of red balls are observed.
    a) Construct the probability distribution of red balls.
    b) Compute the expected number of red balls.
    c) Sketch the probability distribution of red balls.

18. A box contains 10 items of which 3 are defective. 3 items are selected at random without replacement, and number of defective items are observed.
    a) Construct the probability distribution of defective items.
    b) Compute the expected number of defective items.
    c) Sketch the probability distribution of defective items.

19. 3 cards are drawn in succession from a pack of 52 playing cards, without replacement. Find the
    a) probability distribution for number of spades.
    b) expected number of spades.

20. Find the probability distribution for the number of jazz CDs when 4 CDs are selected at random from a collection consisting 5 jazz CDs, 2 classical CDs and 3 rock CDs.

21. Determine the value of $c$, so that the following functions can serve as a probability distribution of the discrete random variable $X$.
    a) $f(x) = c(x^2 + 4)$, for $x = 0, 1, 2, 3$.
    b) $f(x) = c \left( \frac{2}{3} \right)^{3-x}$, for $x = 0, 1, 2$.

22. Find the expectation (mean) $\mu$, variance $\sigma^2$, and standard deviation $\sigma$.
    a) $x_i$ 2 3 8
        $f(x_i)$ 1/4 1/2 1/4
    b) $x_i$ -2 -1 7
        $f(x_i)$ 1/3 1/2 1/6
    c) $x_i$ -1 0 1 2 3
        $f(x_i)$ 0.3 0.1 0.1 0.3 0.2

23. A pair of fair dice is thrown. Let $X$ be the random variable defined by $X(a,b) = \min(a,b)$.
    Find the distribution $f(x)$, mean $\mu$, variance $\sigma^2$, and standard deviation $\sigma$, of $X$.

24. Two cards are selected at random from a box which contains five cards numbered 1, 1, 2, 2, 3. Let $X$ denote the sum and $Y$ denote the maximum of the two numbers drawn. Find the probability distribution and mean of
    a) $X$        b) $Y$        c) $X + Y$

25. A box contains 8 items of which 2 are defective. A man selects 3 items from the box. Find the expected number of defective items he has drawn.

26. A fair coin is tossed four times. Let $X$ be the ‘number of heads’ occurring. Find the distribution, mean, variance and standard deviation of $X$.

27. A fair coin is tossed until a head or four tails occur. Find the expected number of tosses of the coin.

28. A weighted (P(H) = 1/3, and P(T) = 2/3) coin is tossed until a head or five tail occurs. Find the expected number of tosses of the coin.