For the following questions **show all your work clearly** to find the answer.

**Question1.** (pts.) The solution of a system by inverse matrix method is \( X = \begin{pmatrix} 1 & 2 \\ 8 & 4 \end{pmatrix} \).

Obtain the solution set and construct the original system of equations and defined by this solution.

**Question2.** (pts.) The demand function for a monopolist’s product is 
\( p = 400 - 2q \)
and the average cost per unit for producing \( q \) units is 
\[ \bar{C} = q + 160 + \frac{2000}{q} \]
Where \( p \) and \( \bar{C} \) are in dollars per unit. Find the maximum profit that the monopolist can achieve.

**Question3.** (pts.) Determine the following indefinite integrals.

a) \[ \int \frac{2x^4 + 3x^3 - x^2}{x^3} \, dx \]

b) \[ \int \left( e^x + x^e + e^x + e^x \right) \, dx \]

**Question4.** (pts.) Evaluate the definite integral \[ \int_1^3 (x + 3)^2 \, dx \].
Question 5. (pts.) The demand equation for a product \( q = 400 - p^2 \) and the supply equation is \( p = \frac{q}{60} + 5 \).

a) Find an equilibrium point and intercepts on the given graph.

b) Determine Consumer’s surplus and Producer’s surplus under the market equilibrium.

Question 6. (pts.) A monopolist sells two competitive products, A and B, for which the demand functions are

\[
\begin{align*}
 p_A &= 35 - 2q_A^2 + 4q_B, \\
 p_B &= 20 - q_B + q_A
\end{align*}
\]

If the cost is \( C = -8 - 2q_A^3 + 3q_Aq_B + 30q_A + 12q_B + \frac{1}{2}q_A^2 \), how many units of A and B should be sold to maximize the monopolist’s profit?

Question 7. (pts.) The production function for a firm is \( f(l, k) = 12l + 20k - l^2 - 2k^2 \). The cost to the firm of \( l \) and \( k \) is 4 and 8 per unit, respectively. The firm wants the total cost of input to be 88.

a) Construct the model.

b) What will be the expected change in output if the cost is changed from 88 to 89?

Question 8. (pts.) It is estimated that \( x \) months from now the population \( P(x) \) of a certain town will be changing at a rate of \( 2 + 6\sqrt{x} \) people per month. The current population is 5000. What will be the population 9 months from now?

Question 9. (pts.) The marginal profit (derivative of total profit) of a certain company is \( 100 - 2q \) dollars per unit when \( q \) units are produced. If the company’s profit is $700 when 10 units are produced, what is the company’s maximum possible profit?