**Question 1.** (20 points) Solve the given system of linear equations using the Gaussian-Jordan Elimination method

\[
\begin{align*}
2x_1 + 3x_2 &= 10 \\
3x_1 + 4x_2 + 5x_3 &= 5 \\
-x_1 + 2x_2 - 3x_3 &= 10 \\
x_3 + 2x_4 &= -3 \\
x_4 - 4x_5 &= 7 \\
\end{align*}
\]

**Question 2.** (16 points)

\[
A = \begin{pmatrix} 1 & 0 & -1 & 7 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 1 & 2 \end{pmatrix}, \quad \text{where } E_1 \text{ and } E_2 \text{ are the elementary matrices and } R \text{ is the reduced row-echelon form of } A.
\]

a) Find the matrix \( R \) such that \( R = E_2 E_1 A \).

b) Find the matrices \( F \) and \( G \) such that \( A = FGR \).

**Question 3.** (16 points)

If \( A^{-1} = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{pmatrix} \), find the matrix \( A \) using the cofactor matrix.

**Question 4.** (16 points)

a) Suppose that \( A^2 = A \). Show that \( \det(A) = 0 \), or \( \det(A) = 1 \).

b) Suppose that \( A^T = A^{-1} \). Show that \( \det(A) = 1 \), or \( \det(A) = -1 \).
Question 5. (16 points)

a) If $A$ and $B$ are square matrices of the same size and $\det(A) = 2$ and $\det(B) = 3$ then find $\det(A^2B^{-1})$

b) Compute $\det\begin{pmatrix} a & b & c \\ a+x & b+x & c+x \\ a+y & b+y & c+y \end{pmatrix}$

Question 6. (16 points) Consider the following system of linear equations

\[
\begin{align*}
    x_1 + 3x_4 &= 1 \\
    x_2 - 2x_3 &= 2 \\
    -2x_1 + 3x_2 - 2x_3 + 3x_4 &= 3 \\
    -3x_1 + 3x_3 + 3x_4 &= 4
\end{align*}
\]

Find only $x_4$ using Cramer’s Rule.