MATH152 CALCULUS II TUTORIAL – VI

(29.03.2019)

Question 1:

Find the domain of the vector-valued function.

$\mathbf{r}(t) = \ln t\,\mathbf{i} - e^t\mathbf{j} - t\mathbf{k}$

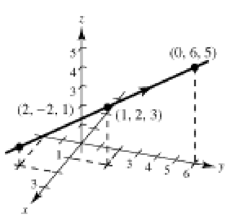
1. Component functions: $f(t) = \ln t$
2. $g(t) = -e^t$
3. $h(t) = -t$
4. Domain: $(0, \infty)$

Question 2 :

Sketch the curve represented by the vector-valued function and give the orientation of the curve.

$\mathbf{r}(t) = (-t + 1)\mathbf{i} + (4t + 2)\mathbf{j} + (2t + 3)\mathbf{k}$

1. $x = -t + 1$
2. $y = 4t + 2$
3. $z = 2t + 3$
4. Line passing through the points:

   $(0, 6, 5)$, $(1, 2, 3)$
5. 

Question 3 :

Represent the plane curve by a vector-valued function. (There are many correct answers.)

$y = (x - 2)^2$

1. Let $x = t$,
2. then $y = (t - 2)^2$.
3. $\mathbf{r}(t) = t\mathbf{i} + (t - 2)^2\,\mathbf{j}$

Question 4:

Find (a) $\mathbf{r}''(t)$ and (b) $\mathbf{r}'(t) \cdot \mathbf{r}''(t)$.

$\mathbf{r}(t) = t^3\mathbf{i} + \frac{1}{2}t^2\mathbf{j}$

1. (a) $\mathbf{r}'(t) = 3t^2\mathbf{i} + t\mathbf{j}$
2. $\mathbf{r}''(t) = 6t\,\mathbf{i} + \mathbf{j}$
3. (b) $\mathbf{r}'(t) \cdot \mathbf{r}''(t) = 3t^2(6t) + t$
4. $= 18t^3 + t$

Question 5:

Evaluate the definite integral.

$\int_0^2 (t\mathbf{i} + e^t\mathbf{j} - te^t\mathbf{k})\,dt$

1. $\int_0^2 (t\mathbf{i} + e^t\mathbf{j} - te^t\mathbf{k})\,dt = \left[\frac{t^2}{2}\mathbf{i}\right]_0^2 + \left[e^t\mathbf{j}\right]_0^2 - \left[(t-1)e^t\mathbf{k}\right]_0^2$
2. $= 2\mathbf{i} + (e^2 - 1)\mathbf{j} - (e^2 + 1)\mathbf{k}$

Question 6:

Evaluate the limit.

$\lim_{t\to 2}\left(t\mathbf{i} + \frac{t^2 - 4}{t^2 - 2t}\mathbf{j} + \frac{1}{t}\mathbf{k}\right)$

1. $\lim_{t\to 2}\left[t\mathbf{i} + \frac{t^2 - 4}{t^2 - 2t}\mathbf{j} + \frac{1}{t}\mathbf{k}\right] = 2\mathbf{i} + 2\mathbf{j} + \frac{1}{2}\mathbf{k}$
2. since

   $\lim_{t\to 2}\frac{t^2 - 4}{t^2 - 2t} = \lim_{t\to 2}\frac{2t}{2t - 2}$
3. $= 2.$ (L'Hôpital's Rule)

Question : 7

Evaluate the limit.

$$\lim_{t \to 0}\left(t^2\mathbf{i} + 3t\mathbf{j} + \frac{1-\cos t}{t}\mathbf{k}\right)$$

1. $\lim_{t \to 0}\left[t^2\mathbf{i} + 3t\mathbf{j} + \frac{1-\cos t}{t}\mathbf{k}\right] = \mathbf{0}$

2. since

$$\lim_{t \to 0}\frac{1-\cos t}{t} = \lim_{t \to 0}\frac{\sin t}{1}$$

3. $= 0.$ (L'Hôpital's Rule)

Question 8:

Find the unit tangent vector $\mathbf{T}(t)$ and find a set of parametric equations for the line tangent to the space curve at point P.

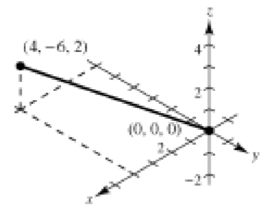
$\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t\mathbf{k}, \quad P(0, 0, 0)$

1. $\mathbf{r}'(t) = \mathbf{i} + 2t\mathbf{j} + \mathbf{k}$
2. When $t = 0$, $\mathbf{r}'(0) = \mathbf{i} + \mathbf{k}$,
3. $[t = 0 \text{ at } (0, 0, 0)]$.
4. $\mathbf{T}(0) = \frac{\mathbf{r}'(0)}{\|\mathbf{r}'(0)\|}$
5. $= \frac{\sqrt{2}}{2}(\mathbf{i} + \mathbf{k})$
6. Direction numbers: $a = 1$,
7. $b = 0$,
8. $c = 1$
9. Parametric equations: $x = t$,
10. $y = 0$,
11. $z = t$

Question 9:

Sketch the space curve and find its length over the given interval.

Function	*Interval*
$\mathbf{r}(t) = 2t\mathbf{i} - 3t\mathbf{j} + t\mathbf{k}$	$[0, 2]$

1. 

2. $\frac{dx}{dt} = 2$

3. $\frac{dy}{dt} = -3$

4. $\frac{dz}{dt} = 1$

5. $s = \int_0^2 \sqrt{2^2 + (-3)^2 + 1^2}\,dt$

6. $= \int_0^2 \sqrt{14}\,dt$

7. $= \left[\sqrt{14}\,t\right]_0^2$

8. $= 2\sqrt{14}$

Question 10:

Find and simplify the function values.

$f(x, y) = x \sin y$

(a) $\left(2, \frac{\pi}{4}\right)$ (b) $(3, 1)$ (c) $\left(-3, \frac{\pi}{3}\right)$ (d) $\left(4, \frac{\pi}{2}\right)$

1. (a) $f\left(2, \frac{\pi}{4}\right) = 2 \sin \frac{\pi}{4}$
2. $= \sqrt{2}$
3. (b) $f(3, 1) = 3 \sin(1)$
4. (c) $f\left(-3, \frac{\pi}{3}\right) = -3 \sin \frac{\pi}{3}$
5. $= -3\left(\frac{\sqrt{3}}{2}\right)$
6. $= \frac{-3\sqrt{3}}{2}$
7. (d) $f\left(4, \frac{\pi}{2}\right) = 4 \sin \frac{\pi}{2}$
8. $= 4$

Question 11:

Describe the domain and range of the function.

$z = \frac{x + y}{xy}$

1. Domain: $\{(x, y): x \neq 0 \text{ and } y \neq 0\}$
2. Range: all real numbers

Question 12

Describe the domain and range of the function.

$f(x, y) = \ln(4 - x - y)$

1. Domain: $4 - x - y > 0$
2. $x + y < 4$
3. $\{(x, y): y < -x + 4\}$
4. Range: all real numbers

Question 13:

Describe the domain and range of the function.

$f(x, y) = \ln(4 - x - y)$

1. Domain: $4 - x - y > 0$
2. $x + y < 4$
3. $\{(x, y): y < -x + 4\}$
4. Range: all real numbers

Question 14:

Find the limit (if it exists). If the limit does not exist, explain why.

$\lim_{(x, y)\to(1, 1)} \frac{xy - 1}{1 + xy}$

1. $\lim_{(x, y)\to(1, 1)} \frac{xy - 1}{1 + xy} = \frac{1 - 1}{1 + 1}$
2. $= 0$

Question 15:

Find the limit and discuss the continuity of the function (if it exists).

$\lim_{(x, y)\to(0, 0)} \frac{-4x^2y}{x^4 + y^2}$

1. For $y = x^2$, $\frac{-4x^2y}{x^4 + y^2} = \frac{-4x^4}{x^4 + x^4}$
2. $= -2$, for $x \neq 0$
3. For $y = 0$, $\frac{-4x^2y}{x^4 + y^2} = 0$, for $x \neq 0$
4. Thus, the limit does not exist.
5. Continuous except at $(0, 0)$.

Question 16:

Find the limit and discuss the continuity of the function.

$\lim_{(x, y)\to(-1, 2)} e^{xy}$

1. $\lim_{(x, y)\to(-1, 2)} e^{xy} = e^{-2}$
2. $= \frac{1}{e^2}$
3. Continuous everywhere