MATH152 CALCULUS II   TUTORIAL – VI

(01.04.2016)

Question 1:

Find the domain of the vector-valued function.

$\mathbf{r}(t) = \ln t\mathbf{i} - e^t\mathbf{j} - t\mathbf{k}$

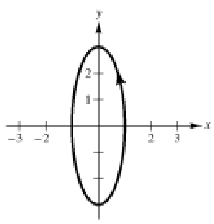
1. Component functions: $f(t) = \ln t$
2. $g(t) = -e^t$
3. $h(t) = -t$
4. Domain: $(0, \infty)$

Question 2 :

Sketch the curve represented by the vector-valued function and give the orientation of the curve.

$\mathbf{r}(\theta) = \cos\theta\mathbf{i} + 3\sin\theta\mathbf{j}$

1. $x = \cos\theta$
2. $y = 3\sin\theta$
3. $x^2 + \frac{y^2}{9} = 1$
4. Ellipse
5.



Question 3 :

Represent the plane curve by a vector-valued function. (There are many correct answers.)

$y = (x - 2)^2$

1. Let $x = t$,
2. then $y = (t - 2)^2$.
3. $\mathbf{r}(t) = t\mathbf{i} + (t - 2)^2\mathbf{j}$

Question 4:

Find (a) $\mathbf{r}''(t)$ and (b) $\mathbf{r}'(t) \cdot \mathbf{r}''(t)$.

$\mathbf{r}(t) = \frac{1}{2}t^2\mathbf{i} - t\mathbf{j} + \frac{1}{6}t^3\mathbf{k}$

1. (a) $\mathbf{r}'(t) = t\mathbf{i} - \mathbf{j} + \frac{1}{2}t^2\mathbf{k}$
2. $\mathbf{r}''(t) = \mathbf{i} + t\mathbf{k}$
3. (b) $\mathbf{r}'(t) \cdot \mathbf{r}''(t) = t(1) - 1(0) + \frac{1}{2}t^2(t)$
4. $= t + \frac{t^3}{2}$

Question 5:

Evaluate the definite integral.

$$\int_0^{\pi/2} [(a\cos t)\mathbf{i} + (a\sin t)\mathbf{j} + \mathbf{k}]\, dt$$

1. $\int_0^{\pi/2} [(a\cos t)\mathbf{i} + (a\sin t)\mathbf{j} + \mathbf{k}]\, dt = \Big[a\sin t\mathbf{i}\Big]_0^{\pi/2} - \Big[a\cos t\mathbf{j}\Big]_0^{\pi/2} + \Big[t\mathbf{k}\Big]_0^{\pi/2}$
2. $= a\mathbf{i} + a\mathbf{j} + \frac{\pi}{2}\mathbf{k}$

Question 6:

Evaluate the limit.

$$\lim_{t\to 0}\left(t^2\mathbf{i} + 3t\mathbf{j} + \frac{1-\cos t}{t}\mathbf{k}\right)$$

1. $\lim_{t\to 0}\left[t^2\mathbf{i} + 3t\mathbf{j} + \frac{1-\cos t}{t}\mathbf{k}\right] = \mathbf{0}$
2. since

   $\lim_{t\to 0}\frac{1-\cos t}{t} = \lim_{t\to 0}\frac{\sin t}{1}$
3. $= 0.$ (L'Hôpital's Rule)

Question : 7

Question 8:

Find the unit tangent vector to the curve given by

$\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j}$

when $t = 1$.

Solution The derivative of $\mathbf{r}(t)$ is

$\mathbf{r}'(t) = \mathbf{i} + 2t\mathbf{j}$. Derivative of $\mathbf{r}(t)$

So, the unit tangent vector is

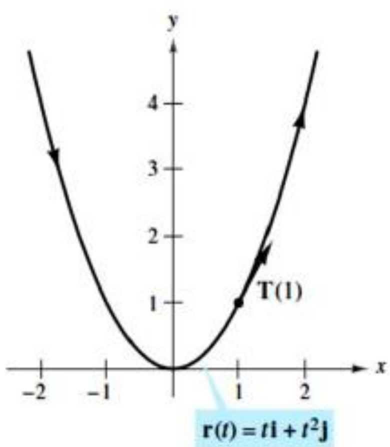
$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$$ Definition of $\mathbf{T}(t)$

$$= \frac{1}{\sqrt{1+4t^2}}(\mathbf{i} + 2t\mathbf{j}).$$ Substitute for $\mathbf{r}'(t)$.

When $t = 1$, the unit tangent vector is

$$\mathbf{T}(1) = \frac{1}{\sqrt{5}}(\mathbf{i} + 2\mathbf{j})$$

as shown in Figure 12.20.

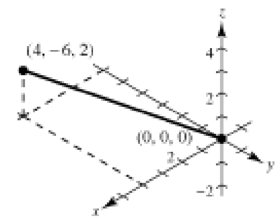


The direction of the unit tangent vector depends on the orientation of the curve.
Figure 12.20

Sketch the space curve and find its length over the given interval.

Function	*Interval*
$\mathbf{r}(t) = 2t\mathbf{i} - 3t\mathbf{j} + t\mathbf{k}$	$[0, 2]$

1.



2. $\frac{dx}{dt} = 2$

3. $\frac{dy}{dt} = -3$

4. $\frac{dz}{dt} = 1$

5. $s = \int_0^2 \sqrt{2^2 + (-3)^2 + 1^2}\, dt$

6. $= \int_0^2 \sqrt{14}\, dt$

7. $= \left[\sqrt{14}\,t\right]_0^2$

8. $= 2\sqrt{14}$

Question 9:

Find and simplify the function values.

$f(x, y) = xe^y$

(a) $(5, 0)$ (b) $(3, 2)$ (c) $(2, -1)$

(d) $(5, y)$ (e) $(x, 2)$ (f) (t, t)

1. (a) $f(5, 0) = 5e^0$
2. $= 5$
3. (b) $f(3, 2) = 3e^2$
4. (c) $f(2, -1) = 2e^{-1}$
5. $= \dfrac{2}{e}$
6. (d) $f(5, y) = 5e^y$
7. (e) $f(x, 2) = xe^2$
8. (f) $f(t, t) = te^t$

Question 10:

Describe the domain and range of the function.

$f(x, y) = \ln(4 - x - y)$

1. Domain: $4 - x - y > 0$
2. $x + y < 4$
3. $\{(x, y): y < -x + 4\}$
4. Range: all real numbers

Question 11

Describe the domain and range of the function.

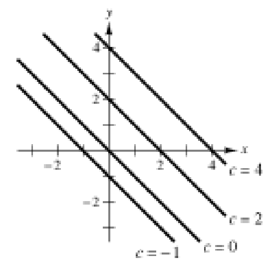
$f(x, y) = \sqrt{4 - x^2 - y^2}$

1. Domain: $4 - x^2 - y^2 \geq 0$
2. $x^2 + y^2 \leq 4$
3. $\{(x, y): x^2 + y^2 \leq 4\}$
4. Range: $0 \leq z \leq 2$

Question 12:

Describe the level curves of the function. Sketch the level curves for the given c-values.

$z = x + y, \quad c = -1, 0, 2, 4$

1. Level curves are parallel lines of the form $x + y = c$.
2. 

Question 13:

Describe the level curves of the function. Sketch the level curves for the given c-values.

$z = \sqrt{25 - x^2 - y^2}, \quad c = 0, 1, 2, 3, 4, 5$

1. The level curves are of the form

   $$c = \sqrt{25 - x^2 - y^2},$$

2. $x^2 + y^2 = 25 - c^2.$
3. Thus, the level curves are circles of radius 5 or less, centered at the origin.
4.