

## MATH152 CALCULUS II TUTORIAL – VI

(02.11.2018)

### Question 1:

Find the domain of the vector-valued function.

$$\mathbf{r}(t) = \ln t \mathbf{i} - e^t \mathbf{j} - t \mathbf{k}$$

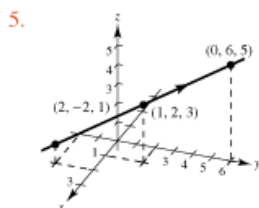
1. Component functions:  $f(t) = \ln t$
2.  $g(t) = -e^t$
3.  $h(t) = -t$
4. Domain:  $(0, \infty)$

### Question 2 :

Sketch the curve represented by the vector-valued function and give the orientation of the curve.

$$\mathbf{r}(t) = (-t + 1)\mathbf{i} + (4t + 2)\mathbf{j} + (2t + 3)\mathbf{k}$$

1.  $x = -t + 1$
2.  $y = 4t + 2$
3.  $z = 2t + 3$
4. Line passing through the points:  
 $(0, 6, 5), (1, 2, 3)$



### Question 3 :

Represent the plane curve by a vector-valued function.  
(There are many correct answers.)

$$y = (x - 2)^2$$

1. Let  $x = t$ ,
2. then  $y = (t - 2)^2$ .
3.  $\mathbf{r}(t) = t \mathbf{i} + (t - 2)^2 \mathbf{j}$

### Question 4:

Find (a)  $\mathbf{r}'(t)$  and (b)  $\mathbf{r}'(t) \cdot \mathbf{r}''(t)$ .

$$\mathbf{r}(t) = t^3 \mathbf{i} + \frac{1}{2}t^2 \mathbf{j}$$

1. (a)  $\mathbf{r}'(t) = 3t^2 \mathbf{i} + t \mathbf{j}$
2.  $\mathbf{r}''(t) = 6t \mathbf{i} + \mathbf{j}$
3. (b)  $\mathbf{r}'(t) \cdot \mathbf{r}''(t) = 3t^2(6t) + t$
4.  $= 18t^3 + t$

### Question 5:

Evaluate the definite integral.

$$\int_0^2 (t \mathbf{i} + e^t \mathbf{j} - te^t \mathbf{k}) dt$$

1.  $\int_0^2 (t \mathbf{i} + e^t \mathbf{j} - te^t \mathbf{k}) dt = \left[ \frac{t^2}{2} \mathbf{i} \right]_0^2 + \left[ e^t \mathbf{j} \right]_0^2 - \left[ (t-1)e^t \mathbf{k} \right]_0^2$
2.  $= 2 \mathbf{i} + (e^2 - 1) \mathbf{j} - (e^2 + 1) \mathbf{k}$

### Question 6:

Evaluate the limit.

$$\lim_{t \rightarrow 2} \left( t \mathbf{i} + \frac{t^2 - 4}{t^2 - 2t} \mathbf{j} + \frac{1}{t} \mathbf{k} \right)$$

1.  $\lim_{t \rightarrow 2} \left[ t \mathbf{i} + \frac{t^2 - 4}{t^2 - 2t} \mathbf{j} + \frac{1}{t} \mathbf{k} \right] = 2 \mathbf{i} + 2 \mathbf{j} + \frac{1}{2} \mathbf{k}$
2. since

$$\lim_{t \rightarrow 2} \frac{t^2 - 4}{t^2 - 2t} = \lim_{t \rightarrow 2} \frac{2t}{2t - 2}$$

3.  $= 2$ . (L'Hôpital's Rule)

### Question : 7

Evaluate the limit.

$$\lim_{t \rightarrow 0} \left( t^2 \mathbf{i} + 3t \mathbf{j} + \frac{1 - \cos t}{t} \mathbf{k} \right)$$

1.  $\lim_{t \rightarrow 0} \left[ t^2 \mathbf{i} + 3t \mathbf{j} + \frac{1 - \cos t}{t} \mathbf{k} \right] = \mathbf{0}$

2. since

$$\lim_{t \rightarrow 0} \frac{1 - \cos t}{t} = \lim_{t \rightarrow 0} \frac{\sin t}{1}$$

3.  $= 0.$  (L'Hôpital's Rule)

### Question 8:

Find the unit tangent vector  $\mathbf{T}(t)$  and find a set of parametric equations for the line tangent to the space curve at point  $P$ .

$$\mathbf{r}(t) = t \mathbf{i} + t^2 \mathbf{j} + t \mathbf{k}, \quad P(0, 0, 0)$$

1.  $\mathbf{r}'(t) = \mathbf{i} + 2t \mathbf{j} + \mathbf{k}$

2. When  $t = 0$ ,  $\mathbf{r}'(0) = \mathbf{i} + \mathbf{k}$ ,

3.  $[t = 0 \text{ at } (0, 0, 0)].$

4.  $\mathbf{T}(0) = \frac{\mathbf{r}'(0)}{\|\mathbf{r}'(0)\|}$

5.  $= \frac{\sqrt{2}}{2}(\mathbf{i} + \mathbf{k})$

6. Direction numbers:  $a = 1$ ,

7.  $b = 0$ ,

8.  $c = 1$

9. Parametric equations:  $x = t$ ,

10.  $y = 0$ ,

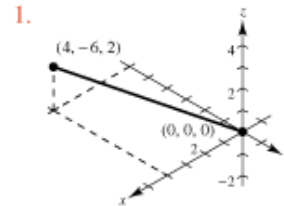
11.  $z = t$

### Question 9:

Sketch the space curve and find its length over the given interval.

<u>Function</u>	<u>Interval</u>
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$\mathbf{r}(t) = 2t \mathbf{i} - 3t \mathbf{j} + t \mathbf{k}$	$[0, 2]$
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2.  $\frac{dx}{dt} = 2$

3.  $\frac{dy}{dt} = -3$

4.  $\frac{dz}{dt} = 1$

5.  $s = \int_0^2 \sqrt{2^2 + (-3)^2 + 1^2} dt$

6.  $= \int_0^2 \sqrt{14} dt$

7.  $= \left[ \sqrt{14} t \right]_0^2$

8.  $= 2\sqrt{14}$

### Question 10:

Find and simplify the function values.

$$f(x, y) = x \sin y$$

(a)  $\left( 2, \frac{\pi}{4} \right)$    (b)  $(3, 1)$    (c)  $\left( -3, \frac{\pi}{3} \right)$    (d)  $\left( 4, \frac{\pi}{2} \right)$

1. (a)  $f\left( 2, \frac{\pi}{4} \right) = 2 \sin \frac{\pi}{4}$

2.  $= \sqrt{2}$

3. (b)  $f(3, 1) = 3 \sin(1)$

4. (c)  $f\left( -3, \frac{\pi}{3} \right) = -3 \sin \frac{\pi}{3}$

5.  $= -3 \left( \frac{\sqrt{3}}{2} \right)$

6.  $= \frac{-3\sqrt{3}}{2}$

7. (d)  $f\left( 4, \frac{\pi}{2} \right) = 4 \sin \frac{\pi}{2}$

8.  $= 4$

**Question 11:**

Describe the domain and range of the function.

$$z = \frac{x + y}{xy}$$

1. Domain:  $\{(x, y): x \neq 0 \text{ and } y \neq 0\}$
2. Range: all real numbers

**Question 12**

Describe the domain and range of the function.

$$f(x, y) = \ln(4 - x - y)$$

1. Domain:  $4 - x - y > 0$
2.  $x + y < 4$
3.  $\{(x, y): y < -x + 4\}$
4. Range: all real numbers

**Question 13:**

Describe the domain and range of the function.

$$f(x, y) = \ln(4 - x - y)$$

1. Domain:  $4 - x - y > 0$
2.  $x + y < 4$
3.  $\{(x, y): y < -x + 4\}$
4. Range: all real numbers

**Question 14:**

Find the limit (if it exists). If the limit does not exist, explain why.

$$\lim_{(x, y) \rightarrow (1, 1)} \frac{xy - 1}{1 + xy}$$

1.  $\lim_{(x, y) \rightarrow (1, 1)} \frac{xy - 1}{1 + xy} = \frac{1 - 1}{1 + 1}$
2.  $= 0$

**Question 15:**

Find the limit and discuss the continuity of the function (if it exists).

$$\lim_{(x, y) \rightarrow (0, 0)} \frac{-4x^2y}{x^4 + y^2}$$

1. For  $y = x^2$ ,  $\frac{-4x^2y}{x^4 + y^2} = \frac{-4x^4}{x^4 + x^4}$
2.  $= -2$ , for  $x \neq 0$
3. For  $y = 0$ ,  $\frac{-4x^2y}{x^4 + y^2} = 0$ , for  $x \neq 0$
4. Thus, the limit does not exist.
5. Continuous except at  $(0, 0)$ .

**Question 16:**

Find the limit and discuss the continuity of the function.

$$\lim_{(x, y) \rightarrow (-1, 2)} e^{xy}$$

1.  $\lim_{(x, y) \rightarrow (-1, 2)} e^{xy} = e^{-2}$
2.  $= \frac{1}{e^2}$
3. Continuous everywhere