

# MATH152 CALCULUS II TUTORIAL – VI

(03.11.2017)

## Question 1:

Find the domain of the vector-valued function.

$$\mathbf{r}(t) = 5t\mathbf{i} - 4t\mathbf{j} - \frac{1}{t}\mathbf{k}$$

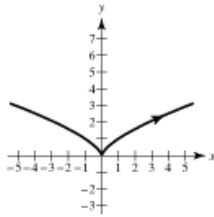
1. Component functions:  $f(t) = 5t$
2.  $g(t) = -4t$
3.  $h(t) = -\frac{1}{t}$
4. Domain:  $(-\infty, 0) \cup (0, \infty)$

## Question 2 :

Sketch the curve represented by the vector-valued function and give the orientation of the curve.

$$\mathbf{r}(t) = t^3\mathbf{i} + t^2\mathbf{j}$$

1.  $x = t^3$
2.  $y = t^2$
3.  $y = x^{2/3}$
- 4.



## Question 3 :

Represent the plane curve by a vector-valued function.  
(There are many correct answers.)

$$y = 4 - x$$

1. Let  $x = t$ ,
2. then  $y = 4 - t$ .
3.  $\mathbf{r}(t) = t\mathbf{i} + (4 - t)\mathbf{j}$

## Question 4:

Find (a)  $\mathbf{r}'(t)$  and (b)  $\mathbf{r}'(t) \cdot \mathbf{r}''(t)$ .

$$\mathbf{r}(t) = t^3\mathbf{i} + \frac{1}{2}t^2\mathbf{j}$$

1. (a)  $\mathbf{r}'(t) = 3t^2\mathbf{i} + t\mathbf{j}$
2.  $\mathbf{r}''(t) = 6t\mathbf{i} + \mathbf{j}$
3. (b)  $\mathbf{r}'(t) \cdot \mathbf{r}''(t) = 3t^2(6t) + t$
4.  $= 18t^3 + t$

## Question 5:

Evaluate the definite integral.

$$\int_0^1 (8t\mathbf{i} + t\mathbf{j} - \mathbf{k}) dt$$

1.  $\int_0^1 (8t\mathbf{i} + t\mathbf{j} - \mathbf{k}) dt = \left[ 4t^2\mathbf{i} \right]_0^1 + \left[ \frac{t^2}{2}\mathbf{j} \right]_0^1 - \left[ t\mathbf{k} \right]_0^1$
2.  $= 4\mathbf{i} + \frac{1}{2}\mathbf{j} - \mathbf{k}$

## Question 6:

Evaluate the limit.

$$\lim_{t \rightarrow 2} \left( t\mathbf{i} + \frac{t^2 - 4}{t^2 - 2t}\mathbf{j} + \frac{1}{t}\mathbf{k} \right)$$

1.  $\lim_{t \rightarrow 2} \left[ t\mathbf{i} + \frac{t^2 - 4}{t^2 - 2t}\mathbf{j} + \frac{1}{t}\mathbf{k} \right] = 2\mathbf{i} + 2\mathbf{j} + \frac{1}{2}\mathbf{k}$
2. since
3.  $\lim_{t \rightarrow 2} \frac{t^2 - 4}{t^2 - 2t} = \lim_{t \rightarrow 2} \frac{2t}{2t - 2} = 2$ . (L'Hôpital's Rule)

### Question 7:

Find the unit tangent vector to the curve at the specified value of the parameter.

$$\mathbf{r}(t) = 4 \cos t \mathbf{i} + 4 \sin t \mathbf{j}, \quad t = \frac{\pi}{4}$$

- $\mathbf{r}'(t) = -4 \sin t \mathbf{i} + 4 \cos t \mathbf{j}$
- $\|\mathbf{r}'(t)\| = \sqrt{16 \sin^2 t + 16 \cos^2 t}$
- $= 4$
- $\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$
- $= -\sin t \mathbf{i} + \cos t \mathbf{j}$
- $\mathbf{T}\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} \mathbf{i} + \frac{\sqrt{2}}{2} \mathbf{j}$

### Question 8:

Find the unit tangent vector  $\mathbf{T}(t)$  and find a set of parametric equations for the line tangent to the space curve at point  $P$ .

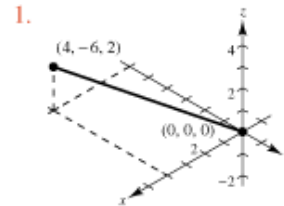
$$\mathbf{r}(t) = t \mathbf{i} + t^2 \mathbf{j} + t \mathbf{k}, \quad P(0, 0, 0)$$

- $\mathbf{r}'(t) = \mathbf{i} + 2t \mathbf{j} + \mathbf{k}$
- When  $t = 0$ ,  $\mathbf{r}'(0) = \mathbf{i} + \mathbf{k}$ ,
- $[t = 0 \text{ at } (0, 0, 0)]$ .
- $\mathbf{T}(0) = \frac{\mathbf{r}'(0)}{\|\mathbf{r}'(0)\|}$
- $= \frac{\sqrt{2}}{2}(\mathbf{i} + \mathbf{k})$
- Direction numbers:  $a = 1$ ,
- $b = 0$ ,
- $c = 1$
- Parametric equations:  $x = t$ ,
- $y = 0$ ,
- $z = t$

### Question 9:

Sketch the space curve and find its length over the given interval.

<u>Function</u>	<u>Interval</u>
$\mathbf{r}(t) = 2t \mathbf{i} - 3t \mathbf{j} + t \mathbf{k}$	$[0, 2]$



- $\frac{dx}{dt} = 2$
- $\frac{dy}{dt} = -3$
- $\frac{dz}{dt} = 1$
- $s = \int_0^2 \sqrt{2^2 + (-3)^2 + 1^2} dt$
- $= \int_0^2 \sqrt{14} dt$
- $= \left[ \sqrt{14} t \right]_0^2$
- $= 2\sqrt{14}$

### Question 10:

Find and simplify the function values.

$$f(x, y) = x \sin y$$

$$(a) \left(2, \frac{\pi}{4}\right) \quad (b) (3, 1) \quad (c) \left(-3, \frac{\pi}{3}\right) \quad (d) \left(4, \frac{\pi}{2}\right)$$

- (a)  $f\left(2, \frac{\pi}{4}\right) = 2 \sin \frac{\pi}{4}$
- $= \sqrt{2}$
- (b)  $f(3, 1) = 3 \sin(1)$
- (c)  $f\left(-3, \frac{\pi}{3}\right) = -3 \sin \frac{\pi}{3}$
- $= -3 \left(\frac{\sqrt{3}}{2}\right)$
- $= \frac{-3\sqrt{3}}{2}$
- (d)  $f\left(4, \frac{\pi}{2}\right) = 4 \sin \frac{\pi}{2}$
- $= 4$

**Question 11:**

Describe the domain and range of the function.

$$z = \frac{x + y}{xy}$$

1. Domain:  $\{(x, y): x \neq 0 \text{ and } y \neq 0\}$
2. Range: all real numbers

**Question 12**

Describe the domain and range of the function.

$$f(x, y) = \sqrt{4 - x^2 - y^2}$$

1. Domain:  $4 - x^2 - y^2 \geq 0$
2.  $x^2 + y^2 \leq 4$
3.  $\{(x, y): x^2 + y^2 \leq 4\}$
4. Range:  $0 \leq z \leq 2$

**Question 13:**

Describe the domain and range of the function.

$$f(x, y) = \ln(4 - x - y)$$

1. Domain:  $4 - x - y > 0$
2.  $x + y < 4$
3.  $\{(x, y): y < -x + 4\}$
4. Range: all real numbers

**Question 14:**

Find the limit (if it exists). If the limit does not exist, explain why.

$$\lim_{(x, y) \rightarrow (1, 1)} \frac{xy - 1}{1 + xy}$$

1.  $\lim_{(x, y) \rightarrow (1, 1)} \frac{xy - 1}{1 + xy} = \frac{1 - 1}{1 + 1}$
2.  $= 0$

**Question 15:**

Find the limit and discuss the continuity of the function (if it exists).

$$\lim_{(x, y) \rightarrow (0, 0)} \frac{-4x^2y}{x^4 + y^2}$$

1. For  $y = x^2$ ,  $\frac{-4x^2y}{x^4 + y^2} = \frac{-4x^4}{x^4 + x^4}$
2.  $= -2$ , for  $x \neq 0$
3. For  $y = 0$ ,  $\frac{-4x^2y}{x^4 + y^2} = 0$ , for  $x \neq 0$
4. Thus, the limit does not exist.
5. Continuous except at  $(0, 0)$ .

**Question 16:**

Find the limit and discuss the continuity of the function.

$$\lim_{(x, y) \rightarrow (-1, 2)} e^{xy}$$

1.  $\lim_{(x, y) \rightarrow (-1, 2)} e^{xy} = e^{-2}$
2.  $= \frac{1}{e^2}$
3. Continuous everywhere