# MATH152 CALCULUS II TUTORIAL – V

**(22.03.2019)**

## Question 1:

Find a set of parametric equations of the line.

The line passes through the point $(2, 1, 2)$ and is parallel to the line $x = -t, y = 1 + t, z = -2 + t$.

1. Point: $(2, 1, 2)$
2. Direction vector: $\langle -1, 1, 1\rangle$
3. Direction numbers: $-1, 1, 1$
4. Parametric: $x = 2 - t, y = 1 + t, z = 2 + t$

## Question 2:

Determine if any of the planes are parallel or identical.

$P_1$: $3x - 2y + 5z = 10$

$P_2$: $-6x + 4y - 10z = 5$

$P_3$: $-3x + 2y + 5z = 8$

$P_4$: $75x - 50y + 125z = 250$

1. $P_1$: $\mathbf{n} = \langle 3, -2, 5\rangle$ $(1, -1, 1)$ on plane
2. $P_2$: $\mathbf{n} = \langle -6, 4, -10\rangle$ $(1, -1, 1)$ not on plane
3. $P_3$: $\mathbf{n} = \langle -3, 2, 5\rangle$
4. $P_4$: $\mathbf{n} = \langle 75, -50, 125\rangle$ $(1, -1, 1)$ on plane
5. $P_1$ and $P_4$ are identical.
6. $P_1 = P_4$ is parallel to $P_2$.

## Question 3 :

Find an equation of the plane.

The plane passes through $(-3, -4, 2)$, $(-3, 4, 1)$, and $(1, 1, -2)$.

1. $P = (-3, -4, 2)$
2. $Q = (-3, 4, 1)$
3. $R = (1, 1, -2)$
4. $\overrightarrow{PQ} = \langle 0, 8, -1\rangle$
5. $\overrightarrow{PR} = \langle 4, 5, -4\rangle$
6. $\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR}$
7. $= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8 & -1 \\ 4 & 5 & -4 \end{vmatrix}$
8. $= -27\mathbf{i} - 4\mathbf{j} - 32\mathbf{k}$
9. $-27(x + 3) - 4(y + 4) - 32(z - 2) = 0$
10. $27x + 4y + 32z = -33$

## Question 4 :

Find an equation of the plane.

The plane contains the lines given by

$$\frac{x-1}{-2} = y - 4 = z \text{ and } \frac{x-2}{-3} = \frac{y-1}{4} = \frac{z-2}{-1}.$$

1. The direction vectors for the lines are $\mathbf{u} = -2\mathbf{i} + \mathbf{j} + \mathbf{k}$,
2. $\mathbf{v} = -3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$.
3. Normal vector: $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 1 & 1 \\ -3 & 4 & -1 \end{vmatrix}$
4. $= -5(\mathbf{i} + \mathbf{j} + \mathbf{k})$
5. Point of intersection of the lines: $(-1, 5, 1)$
6. $(x + 1) + (y - 5) + (z - 1) = 0$
7. $x + y + z = 5$

## Question 5 :

Find an equation of the plane.

The plane passes through the points $(1, -2, -1)$ and $(2, 5, 6)$ and is parallel to the $x$-axis.

1. Let $\mathbf{u} = \mathbf{i}$ and
2. let $\mathbf{v}$ be the vector from $(1, -2, -1)$ to $(2, 5, 6)$:

   $\mathbf{v} = \mathbf{i} + 7\mathbf{j} + 7\mathbf{k}$
3. Since $\mathbf{u}$ and $\mathbf{v}$ both lie in the plane $P$, the normal vector to $P$ is:

   $$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 0 \\ 1 & 7 & 7 \end{vmatrix}$$
4. $= -7\mathbf{j} + 7\mathbf{k}$
5. $= -7(\mathbf{j} - \mathbf{k})$
6. $[y - (-2)] - [z - (-1)] = 0$
7. $y - z = -1$

## Question 6:

Find an equation of the plane.

The plane passes through the points $(2, 2, 1)$ and $(-1, 1, -1)$ and is perpendicular to the plane $2x - 3y + z = 3$.

1. Let $\mathbf{v}$ be the vector from $(-1, 1, -1)$ to $(2, 2, 1)$:

   $\mathbf{v} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$
2. Let $\mathbf{n}$ be a vector normal to the plane $2x - 3y + z = 3$:

   $\mathbf{n} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$
3. Since $\mathbf{v}$ and $\mathbf{n}$ both lie in the plane $P$, the normal vector to $P$ is

   $$\mathbf{v} \times \mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 1 & 2 \\ 2 & -3 & 1 \end{vmatrix}$$
4. $= 7\mathbf{i} + \mathbf{j} - 11\mathbf{k}$

## Question 7:

Find the distance between the point and the plane.

$(0, 0, 0)$, $2x + 3y + z = 12$

1. Point: $Q(0, 0, 0)$
2. Plane: $2x + 3y + z - 12 = 0$
3. Normal to plane: $\mathbf{n} = \langle 2, 3, 1 \rangle$
4. Point in plane: $P(6, 0, 0)$
5. Vector: $\overrightarrow{PQ} = \langle -6, 0\ 0 \rangle$
6. $D = \dfrac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|}$
7. $= \dfrac{|-12|}{\sqrt{14}}$
8. $= \dfrac{6\sqrt{14}}{7}$

## Question 8:

Find the point(s) of intersection (if any) of the plane and the line. Also determine whether the line lies in the plane.

$2x + 3y = 10$, $\dfrac{x-1}{3} = \dfrac{y+1}{-2} = z - 3$

1. Writing the equation of the line in parametric form and substituting into the equation of the plane we have:
2. $x = 1 + 3t$
3. $y = -1 - 2t$
4. $z = 3 + t$
5. $2(1 + 3t) + 3(-1 - 2t) = 10$
6. $-1 = 10$, contradiction
7. Therefore, the line does not intersect the plane.

**Question 9:**

Find a set of parametric equations for the line of intersection of the planes.

$3x + 2y - z = 7$

$x - 4y + 2z = 0$

1. The normals to the planes are $\mathbf{n}_1 = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and
2. $\mathbf{n}_2 = \mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$.
3. The direction vector for the line is

$$\mathbf{n}_2 \times \mathbf{n}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -4 & 2 \\ 3 & 2 & -1 \end{vmatrix}$$

4. $= 7(\mathbf{j} + 2\mathbf{k})$.
5. Now find a point of intersection of the planes.

$6x + 4y - 2z = 14$

6. $x - 4y + 2z = 0$
7. $7x = 14$
8. $x = 2$
9. Substituting 2 for $x$ in the second equation, we have $-4y + 2z = -2$ or $z = 2y - 1$.
10. Letting $y = 1$, a point of intersection is $(2, 1, 1)$.
11. $x = 2$
12. $y = 1 + t$
13. $z = 1 + 2t$

**Question 10**

Find the domain of the vector-valued function.

$\mathbf{r}(t) = \ln t\mathbf{i} - e^t\mathbf{j} - t\mathbf{k}$

1. Component functions: $f(t) = \ln t$
2. $g(t) = -e^t$
3. $h(t) = -t$
4. Domain: $(0, \infty)$

**Question 11**

Evaluate (if possible) the vector-valued function at each given value of $t$.

$\mathbf{r}(t) = \ln t\mathbf{i} + \frac{1}{t}\mathbf{j} + 3t\mathbf{k}$

(a) $\mathbf{r}(2)$ (b) $\mathbf{r}(-3)$ (c) $\mathbf{r}(t - 4)$

***Solution.***

1. (a) $\mathbf{r}(2) = \ln 2\mathbf{i} + \frac{1}{2}\mathbf{j} + 6\mathbf{k}$
2. (b) $\mathbf{r}(-3)$ is not defined. ($\ln(-3)$ does not exist.)
3. (c) $\mathbf{r}(t - 4) = \ln(t - 4)\mathbf{i} + \frac{1}{t - 4}\mathbf{j} + 3(t - 4)\mathbf{k}$