

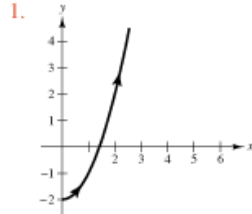
MATH152 CALCULUS II TUTORIAL – 3

(11.10.2019)

Question 1 :

Sketch the curve represented by the parametric equations (indicate the orientation of the curve), and write the corresponding rectangular equation by eliminating the parameter.

$$x = \sqrt{t}, y = t - 2$$



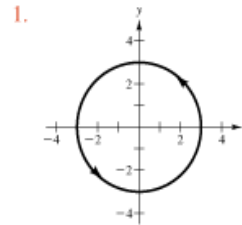
2. $y = x^2 - 2$

3. $x \geq 0$

Question 3:

Sketch the curve represented by the parametric equations (indicate the orientation of the curve), and write the corresponding rectangular equation by eliminating the parameter.

$$x = 3 \cos \theta, y = 3 \sin \theta$$



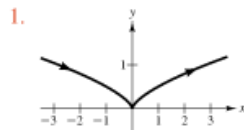
2. Squaring both equations and adding, we have

3. $x^2 + y^2 = 9$.

Question 2:

Sketch the curve represented by the parametric equations (indicate the orientation of the curve), and write the corresponding rectangular equation by eliminating the parameter.

$$x = t^3, y = \frac{t^2}{2}$$



2. $x = t^3$ implies $t = x^{1/3}$

3. $y = \frac{1}{2}x^{2/3}$

Question : 4

Find dy/dx .

$$x = \sin^2 \theta, y = \cos^2 \theta$$

1. $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$

2. $= \frac{-2 \cos \theta \sin \theta}{2 \sin \theta \cos \theta}$

3. $= -1$

4. [Note: $x + y = 1$

5. $\Rightarrow y = 1 - x$

6. and $\frac{dy}{d\theta} = -1$]

Question : 5

Without eliminating the parameter, find dy/dx and d^2y/dx^2 at (1, 1) and (1, -1) on the semicubical parabola given by the parametric equations

$$x = t^2, \quad y = t^3 \quad (-\infty < t < +\infty)$$

Solution

From (4) we have

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2}{2t} = \frac{3}{2}t \quad (t \neq 0) \quad (7)$$

and from (4) applied to $y' = dy/dx$ we have

$$\frac{d^2y}{dx^2} = \frac{dy'}{dx} = \frac{dy'/dt}{dx/dt} = \frac{3/2}{2t} = \frac{3}{4t} \quad (8)$$

Since the point (1, 1) on the curve corresponds to $t = 1$ in the parametric equations, it follows from (7) and (8) that

$$\left. \frac{dy}{dx} \right|_{(1,1)} = \frac{3}{2} \quad \text{and} \quad \left. \frac{d^2y}{dx^2} \right|_{(1,1)} = \frac{3}{4}$$

Question 6 :

Find dy/dx and d^2y/dx^2 , and find the slope and concavity (if possible) at the given value of the parameter.

<u>Parametric Equations</u>	<u>Point</u>
$x = t + 1, \quad y = t^2 + 3t$	$t = -1$

- $\frac{dy}{dx} = \frac{2t + 3}{1}$
- $\frac{dy}{dx} = 1$ when $t = -1$.
- $\frac{d^2y}{dx^2} = 2$
- Concave upwards

Question : 7

Write an integral that represents the arc length of the curve on the given interval. Do not evaluate the integral.

<u>Parametric Equations</u>	<u>Interval</u>
$x = 2t - t^2, \quad y = 2t^{3/2}$	$1 \leq t \leq 2$

- $\frac{dx}{dt} = 2 - 2t$
- $\frac{dy}{dt} = 3t^{1/2}$
- $s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$
- $= \int_1^2 \sqrt{(2 - 2t)^2 + (3t^{1/2})^2} dt$
- $= \int_1^2 \sqrt{4 - 8t + 4t^2 + 9t} dt$
- $= \int_1^2 \sqrt{4t^2 + t + 4} dt$

Question : 8

Find the arc length of the curve on the given interval.

<u>Parametric Equations</u>	<u>Interval</u>
$x = e^{-t} \cos t, \quad y = e^{-t} \sin t$	$0 \leq t \leq \frac{\pi}{2}$

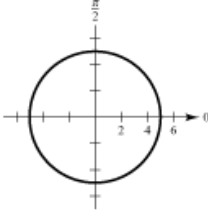
- $\frac{dx}{dt} = -e^{-t}(\sin t + \cos t)$
- $\frac{dy}{dt} = e^{-t}(\cos t - \sin t)$
- $s = \int_0^{\pi/2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$
- $= \int_0^{\pi/2} \sqrt{2e^{-2t}} dt$
- $= -\sqrt{2} \int_0^{\pi/2} e^{-t}(-1) dt$
- $= \left[-\sqrt{2}e^{-t} \right]_0^{\pi/2}$
- $= \sqrt{2}(1 - e^{-\pi/2})$
- ≈ 1.12

Question : 9

Sketch a graph of the polar equation.

$$r = 5$$

1.



2. Circle radius: 5

3. $x^2 + y^2 = 25$

Question : 10

Find (a) $\frac{2}{3}\mathbf{u}$, (b) $\mathbf{v} - \mathbf{u}$, and (c) $2\mathbf{u} + 5\mathbf{v}$.

$$\mathbf{u} = \langle 4, 9 \rangle, \mathbf{v} = \langle 2, -5 \rangle$$

1. (a) $\frac{2}{3}\mathbf{u} = \frac{2}{3}\langle 4, 9 \rangle$

2. $= \langle \frac{8}{3}, 6 \rangle$

3. (b) $\mathbf{v} - \mathbf{u} = \langle 2, -5 \rangle - \langle 4, 9 \rangle$

4. $= \langle -2, -14 \rangle$

5. (c) $2\mathbf{u} + 5\mathbf{v} = 2\langle 4, 9 \rangle + 5\langle 2, -5 \rangle$

6. $= \langle 18, -7 \rangle$

Question 11

Find the following.

(a) $\|\mathbf{u}\|$ (b) $\|\mathbf{v}\|$ (c) $\|\mathbf{u} + \mathbf{v}\|$

(d) $\left\| \frac{\mathbf{u}}{\|\mathbf{u}\|} \right\|$ (e) $\left\| \frac{\mathbf{v}}{\|\mathbf{v}\|} \right\|$ (f) $\left\| \frac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|} \right\|$

$$\mathbf{u} = \langle 1, -1 \rangle, \mathbf{v} = \langle -1, 2 \rangle$$

1. (a) $\|\mathbf{u}\| = \sqrt{1+1}$

2. $= \sqrt{2}$

3. (b) $\|\mathbf{v}\| = \sqrt{1+4}$

4. $= \sqrt{5}$

5. (c) $\mathbf{u} + \mathbf{v} = \langle 0, 1 \rangle$

6. $\|\mathbf{u} + \mathbf{v}\| = \sqrt{0+1}$

7. $= 1$

8. (d) $\frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{1}{\sqrt{2}}\langle 1, -1 \rangle$

9. $\left\| \frac{\mathbf{u}}{\|\mathbf{u}\|} \right\| = 1$

10. (e) $\frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{1}{\sqrt{5}}\langle -1, 2 \rangle$

11. $\left\| \frac{\mathbf{v}}{\|\mathbf{v}\|} \right\| = 1$

12. (f) $\frac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|} = \langle 0, 1 \rangle$

13. $\left\| \frac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|} \right\| = 1$

Question 12

Find the lengths of the sides of the triangle with the indicated vertices, and determine whether the triangle is a right triangle, an isosceles triangle, or neither.

$$(1, -3, -2), (5, -1, 2), (-1, 1, 2)$$

1. $A(1, -3, -2), B(5, -1, 2), C(-1, 1, 2)$

2. $|AB| = \sqrt{16 + 4 + 16}$

3. $= 6$

4. $|AC| = \sqrt{4 + 16 + 16}$

5. $= 6$

6. $|BC| = \sqrt{36 + 4 + 0}$

7. $= 2\sqrt{10}$

8. Since $|AB| = |AC|$, the triangle is isosceles.

Question 13

Find a unit vector (a) in the direction of \mathbf{u} and (b) in the direction opposite of \mathbf{u} .

$$\mathbf{u} = \langle 3, 2, -5 \rangle$$

1. $\|\mathbf{u}\| = \sqrt{9 + 4 + 25}$

2. $= \sqrt{38}$

3. (a) $\frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{1}{\sqrt{38}}\langle 3, 2, -5 \rangle$

4. (b) $-\frac{\mathbf{u}}{\|\mathbf{u}\|} = -\frac{1}{\sqrt{38}}\langle 3, 2, -5 \rangle$

Question 14

Find the vector \mathbf{v} with the given magnitude same direction as \mathbf{u} .

<i>Magnitude</i>	<i>Direction</i>
$\ \mathbf{v}\ = 4$	$\mathbf{u} = \langle 1, 1 \rangle$

1. $\frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{1}{\sqrt{2}}\langle 1, 1 \rangle$

2. $4\left(\frac{\mathbf{u}}{\|\mathbf{u}\|}\right) = 2\sqrt{2}\langle 1, 1 \rangle$

3. $\mathbf{v} = \langle 2\sqrt{2}, 2\sqrt{2} \rangle$