MATH152 CALCULUS II   TUTORIAL – 3

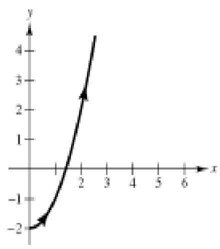
(14.10.2016)

Question 1 :

Sketch the curve represented by the parametric equations (indicate the orientation of the curve), and write the corresponding rectangular equation by eliminating the parameter.

$x = \sqrt{t},\ y = t - 2$

1.



2. $y = x^2 - 2$
3. $x \geq 0$

Question 2:

Find two different sets of parametric equations for the rectangular equation.

$y = x^3$

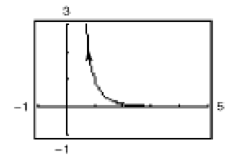
1. Example
2. $x = t, \qquad y = t^3$
3. $x = \sqrt[3]{t}, \qquad y = t$
4. $x = \tan t, \qquad y = \tan^3 t$

Question 3:

Use a graphing utility to graph the curve represented by the parametric equations (indicate the orientation of the curve). Eliminate the parameter and write the corresponding rectangular equation.

$x = e^{-t},\ y = e^{3t}$

1.



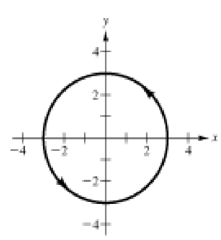
2. $e^t = \dfrac{1}{x}$
3. $e^t = \sqrt[3]{y}$
4. $\sqrt[3]{y} = \dfrac{1}{x}$
5. $y = \dfrac{1}{x^3}$
6. $x > 0$
7. $y > 0$

Question 4:

Sketch the curve represented by the parametric equations (indicate the orientation of the curve), and write the corresponding rectangular equation by eliminating the parameter.

$x = 3\cos\theta,\ y = 3\sin\theta$

1.



2. Squaring both equations and adding, we have
3. $x^2 + y^2 = 9.$

Question : 5

Find dy/dx.

$x = \sin^2 \theta,\ y = \cos^2 \theta$

1. $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$
2. $= \frac{-2\cos\theta \sin\theta}{2\sin\theta\cos\theta}$
3. $= -1$
4. [**Note:** $x + y = 1$
5. $\Rightarrow y = 1 - x$
6. and $\frac{dy}{d\theta} = -1$]

Question : 6

Without eliminating the parameter, find dy/dx and d^2y/dx^2 at $(1, 1)$ and $(1, -1)$ on the semicubical parabola given by the parametric equations

$$x = t^2, \quad y = t^3 \qquad (-\infty < t < +\infty)$$

Solution

From (4) we have

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2}{2t} = \frac{3}{2}t \qquad (t \neq 0) \qquad (7)$$

and from (4) applied to $y' = dy/dx$ we have

$$\frac{d^2y}{dx^2} = \frac{dy'}{dx} = \frac{dy'/dt}{dx/dt} = \frac{3/2}{2t} = \frac{3}{4t} \qquad (8$$

Since the point $(1, 1)$ on the curve corresponds to $t = 1$ in the parametric equations, i follows from (7) and (8) that

$$\left.\frac{dy}{dx}\right|_{t=1} = \frac{3}{2} \quad \text{and} \quad \left.\frac{d^2y}{dx^2}\right|_{t=1} = \frac{3}{4}$$

Question : 7

Write an integral that represents the arc length of the curve on the given interval. Do not evaluate the integral.

Parametric Equations	*Interval*
$x = e^t + 2,\ y = 2t + 1$	$-2 \le t \le 2$

1. $\frac{dx}{dt} = e^t$
2. $\frac{dy}{dt} = 2$
3. $s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$
4. $= \int_{-2}^{2} \sqrt{e^{2t} + 4}\, dt$

Question : 8

Find the arc length of the curve on the given interval.

Parametric Equations	*Interval*
$x = e^{-t}\cos t,\ y = e^{-t}\sin t$	$0 \le t \le \frac{\pi}{2}$

1. $\frac{dx}{dt} = -e^{-t}(\sin t + \cos t)$
2. $\frac{dy}{dt} = e^{-t}(\cos t - \sin t)$
3. $s = \int_0^{\pi/2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$
4. $= \int_0^{\pi/2} \sqrt{2e^{-2t}}\, dt$
5. $= -\sqrt{2}\int_0^{\pi/2} e^{-t}(-1)\, dt$
6. $= \left[-\sqrt{2}e^{-t}\right]_0^{\pi/2}$
7. $= \sqrt{2}(1 - e^{-\pi/2})$
8. ≈ 1.12

Question : 9

Find (a) $\frac{2}{3}\mathbf{u}$, (b) $\mathbf{v} - \mathbf{u}$, and (c) $2\mathbf{u} + 5\mathbf{v}$.

$\mathbf{u} = \langle 4, 9 \rangle,\ \mathbf{v} = \langle 2, -5 \rangle$

1. (a) $\frac{2}{3}\mathbf{u} = \frac{2}{3}\langle 4, 9 \rangle$
2. $= \left\langle \frac{8}{3}, 6 \right\rangle$
3. (b) $\mathbf{v} - \mathbf{u} = \langle 2, -5 \rangle - \langle 4, 9 \rangle$
4. $= \langle -2, -14 \rangle$
5. (c) $2\mathbf{u} + 5\mathbf{v} = 2\langle 4, 9 \rangle + 5\langle 2, -5 \rangle$
6. $= \langle 18, -7 \rangle$

Question 10

Find the following.

(a) $\|\mathbf{u}\|$ (b) $\|\mathbf{v}\|$ (c) $\|\mathbf{u} + \mathbf{v}\|$

(d) $\left\| \frac{\mathbf{u}}{\|\mathbf{u}\|} \right\|$ (e) $\left\| \frac{\mathbf{v}}{\|\mathbf{v}\|} \right\|$ (f) $\left\| \frac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|} \right\|$

$\mathbf{u} = \langle 1, -1 \rangle,\ \mathbf{v} = \langle -1, 2 \rangle$

1. (a) $\|\mathbf{u}\| = \sqrt{1 + 1}$
2. $= \sqrt{2}$
3. (b) $\|\mathbf{v}\| = \sqrt{1 + 4}$
4. $= \sqrt{5}$
5. (c) $\mathbf{u} + \mathbf{v} = \langle 0, 1 \rangle$
6. $\|\mathbf{u} + \mathbf{v}\| = \sqrt{0 + 1}$
7. $= 1$
8. (d) $\frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{1}{\sqrt{2}}\langle 1, -1 \rangle$
9. $\left\| \frac{\mathbf{u}}{\|\mathbf{u}\|} \right\| = 1$
10. (e) $\frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{1}{\sqrt{5}}\langle -1, 2 \rangle$
11. $\left\| \frac{\mathbf{v}}{\|\mathbf{v}\|} \right\| = 1$
12. (f) $\frac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|} = \langle 0, 1 \rangle$
13. $\left\| \frac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|} \right\| = 1$

Question 11

Find the lengths of the sides of the triangle with the indicated vertices, and determine whether the triangle is a right triangle, an isosceles triangle, or neither.

$(1, -3, -2), (5, -1, 2), (-1, 1, 2)$

1. $A(1, -3, -2), B(5, -1, 2), C(-1, 1, 2)$
2. $|AB| = \sqrt{16 + 4 + 16}$
3. $= 6$
4. $|AC| = \sqrt{4 + 16 + 16}$
5. $= 6$
6. $|BC| = \sqrt{36 + 4 + 0}$
7. $= 2\sqrt{10}$
8. Since $|AB| = |AC|$, the triangle is isosceles.

Question 12

Find a unit vector (a) in the direction of $\mathbf{u}$ and (b) in the direction opposite of $\mathbf{u}$.

$\mathbf{u} = \langle 3, 2, -5 \rangle$

1. $\|\mathbf{u}\| = \sqrt{9 + 4 + 25}$
2. $= \sqrt{38}$
3. (a) $\frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{1}{\sqrt{38}}\langle 3, 2, -5 \rangle$
4. (b) $-\frac{\mathbf{u}}{\|\mathbf{u}\|} = -\frac{1}{\sqrt{38}}\langle 3, 2, -5 \rangle$

Question 13

Find the vector $\mathbf{v}$ with the given magnitude
same direction as $\mathbf{u}$.

Magnitude	*Direction*
$\|\mathbf{v}\| = 4$	$\mathbf{u} = \langle 1, 1 \rangle$

1. $\frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{1}{\sqrt{2}}\langle 1, 1 \rangle$
2. $4\left(\frac{\mathbf{u}}{\|\mathbf{u}\|}\right) = 2\sqrt{2}\langle 1, 1 \rangle$
3. $\mathbf{v} = \left\langle 2\sqrt{2}, 2\sqrt{2} \right\rangle$