

MATH152 CALCULUS II TUTORIAL – I

(26.02.2016)

Question 1 : (sequences)

Find the limit (if possible) of the sequence.

$$a_n = \frac{2n}{\sqrt{n^2 + 1}}$$

1. $\lim_{n \rightarrow \infty} \frac{2n}{\sqrt{n^2 + 1}} = \lim_{n \rightarrow \infty} \frac{2}{\sqrt{1 + (1/n^2)}}$

2. $= \frac{2}{1}$

3. $= 2$

Question 2 : (sequences)

Determine the convergence or divergence of the sequence with the given n th term. If the sequence converges, find its limit.

$$a_n = \frac{3^n}{4^n}$$

1. $\lim_{n \rightarrow \infty} \left(\frac{3}{4}\right)^n = 0$

2. Converges

Question 3 : (sequences)

Determine the convergence or divergence of the sequence with the given n th term. If the sequence converges, find its limit.

$$a_n = \frac{(n+1)!}{n!}$$

1. $\lim_{n \rightarrow \infty} \frac{(n+1)!}{n!} = \lim_{n \rightarrow \infty} (n+1)$

2. $= \infty$

3. Diverges

Question 4: (Geometric Series)

Determine the convergence or divergence of the series.

$$\sum_{n=0}^{\infty} \frac{4}{2^n}$$

1. $\sum_{n=0}^{\infty} \frac{4}{2^n} = 4 \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$

2. Geometric series with $r = \frac{1}{2}$

3. Converges by Theorem 9.6

Question 5: (Geometric Series)

Find the sum of the convergent series.

$$3 - 1 + \frac{1}{3} - \frac{1}{9} + \dots$$

1. $\sum_{n=0}^{\infty} 3\left(-\frac{1}{3}\right)^n = \frac{3}{1 - (-1/3)}$

2. $= \frac{9}{4}$

Question 6:

Find the sum of the convergent series.

$$\sum_{n=1}^{\infty} \frac{8}{(n+1)(n+2)}$$

1. $\sum_{n=1}^{\infty} \frac{8}{(n+1)(n+2)} = 8 \sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+2}\right)$

2. $= 8 \left[\left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \dots \right]$

3. $= 8\left(\frac{1}{2}\right)$

4. $= 4$

Question 7: (n'th term test)

Verify that the infinite series diverges.

$$\sum_{n=1}^{\infty} \frac{n}{n+1}$$

1. $\lim_{n \rightarrow \infty} \frac{n}{n+1} = 1$
2. $\neq 0$
3. Diverges by Theorem 9.9

Question 8: (n'th term test)

Determine the convergence or divergence of the series.

$$\sum_{n=2}^{\infty} \frac{n}{\ln n}$$

1. Since $n > \ln(n)$, the terms $a_n = \frac{n}{\ln(n)}$ do not approach 0 as $n \rightarrow \infty$.
2. Hence, the series $\sum_{n=2}^{\infty} \frac{n}{\ln(n)}$ diverges.

Question 9 (p-test)

determine the convergence or divergence of the p-series.

$$1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \frac{1}{5\sqrt{5}} + \dots$$

1. $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$
2. Convergent p-series with $p = \frac{3}{2} > 1$

Question : 10 (Comparison test)

Use the Direct Comparison Test to determine the convergence or divergence of the series.

$$\sum_{n=0}^{\infty} \frac{1}{3^n + 1}$$

1. $0 < \frac{1}{3^n + 1} < \frac{1}{3^n}$
2. Therefore,

$$\sum_{n=0}^{\infty} \frac{1}{3^n + 1}$$

converges by comparison with the convergent geometric series

$$\sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n.$$

Question 11 (Limit comparison test)

Use the Limit Comparison Test to determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \frac{2n^2 - 1}{3n^5 + 2n + 1}$$

1. $\lim_{n \rightarrow \infty} \frac{\frac{2n^2 - 1}{3n^5 + 2n + 1}}{1/n^3} = \lim_{n \rightarrow \infty} \frac{2n^5 - n^3}{3n^5 + 2n + 1}$
2. $= \frac{2}{3}$

3. Therefore,

$$\sum_{n=1}^{\infty} \frac{2n^2 - 1}{3n^5 + 2n + 1}$$

converges by a limit comparison with the convergent p-series

$$\sum_{n=1}^{\infty} \frac{1}{n^3}.$$

Question 12 (Ratio Test)

Use the Ratio Test to determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \frac{2^n}{n^2}$$

1. $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{2^{n+1}}{(n+1)^2} \cdot \frac{n^2}{2^n} \right|$
2. $= \lim_{n \rightarrow \infty} \frac{2n^2}{(n+1)^2}$
3. $= 2$
4. Therefore, by the Ratio Test, the series diverges.

Question 13 (Ratio test)

Determine the convergence or divergence of the series using any appropriate test from this chapter. Identify the test used.

$$\sum_{n=1}^{\infty} \frac{n7^n}{n!}$$

1. $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(n+1)7^{n+1}}{(n+1)!} \cdot \frac{n!}{n7^n} \right|$
2. $= \lim_{n \rightarrow \infty} \frac{7}{n}$
3. $= 0$
4. Therefore, by the Ratio Test, the series converges.

Question 14 (Alternating series Test)

Determine the convergence or divergence of the series using any appropriate test from this chapter. Identify the test used.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 5}{n}$$

1. $a_{n+1} = \frac{5}{n+1}$
2. $< \frac{5}{n}$
3. $= a_n$
4. $\lim_{n \rightarrow \infty} \frac{5}{n} = 0$
5. Therefore, by the Alternating Series Test, the series converges (conditional convergence).

Question 15 (Alternating series Test)

Determine the convergence or divergence of the series.

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$$

1. $a_{n+1} = \frac{1}{(n+1)!}$
2. $< \frac{1}{n!}$
3. $= a_n$
4. $\lim_{n \rightarrow \infty} \frac{1}{n!} = 0$
5. Converges by Theorem 9.14