MATH152 CALCULUS II TUTORIAL – I

(24.02.2017)

Question 1 : (sequences)

Find the limit (if possible) of the sequence.

$$a_n = \frac{5n^2}{n^2 + 2}$$

1. $\lim_{n\to\infty} \frac{5n^2}{n^2 + 2} = 5$

Question 2 : (sequences)

Determine the convergence or divergence of the sequence with the given nth term. If the sequence converges, find its limit.

$$a_n = \frac{\ln(n^3)}{2n}$$

1. $\lim_{n\to\infty} \frac{\ln(n^3)}{2n} = \lim_{n\to\infty} \frac{3}{2}\frac{\ln(n)}{n}$
2. $\quad = \lim_{n\to\infty} \frac{3}{2}\left(\frac{1}{n}\right)$
3. $\quad = 0$
4. Converges
5. (L'Hôpital's Rule)

Question 3 :

Find the sum of the convergent series.

$$\sum_{n=1}^{\infty} \frac{8}{(n+1)(n+2)}$$

1. $\sum_{n=1}^{\infty} \frac{8}{(n+1)(n+2)} = 8\sum_{n=1}^{\infty}\left(\frac{1}{n+1} - \frac{1}{n+2}\right)$
2. $\quad = 8\left[\left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \cdots\right]$
3. $\quad = 8\left(\frac{1}{2}\right)$
4. $\quad = 4$

Note: Find the form of s_n .

Question 4: (Geometric Series)

Determine the convergence or divergence of the series.

$$\sum_{n=0}^{\infty} \frac{4}{2^n}$$

1. $\sum_{n=0}^{\infty} \frac{4}{2^n} = 4\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n$
2. Geometric series with $r = \frac{1}{2}$
3. Converges by Theorem 9.6

Question 5: (Geometric Series)

Find the sum of the convergent series.

$$3 - 1 + \frac{1}{3} - \frac{1}{9} + \cdots$$

1. $\sum_{n=0}^{\infty} 3\left(-\frac{1}{3}\right)^n = \frac{3}{1 - (-1/3)}$
2. $\quad = \frac{9}{4}$

Question 6:

Find the sum of the convergent series.

$$\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n$$

1. $\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n = \frac{1}{1 - (1/2)}$
2. $\quad = 2$

Question 7: (n'th term test)

Determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \frac{3n-1}{2n+1}$$

1. $\lim_{n\to\infty} \frac{3n-1}{2n+1} = \frac{3}{2}$
2. $\neq 0$
3. Diverges

Question 8: (n'th term test)

Determine the convergence or divergence of the series.

$$\sum_{n=2}^{\infty} \frac{n}{\ln n}$$

1. Since $n > \ln(n)$, the terms $a_n = \frac{n}{\ln(n)}$ do not approach 0 as $n \to \infty$.
2. Hence, the series $\sum_{n=2}^{\infty} \frac{n}{\ln(n)}$ diverges.

Question : 9 (p-test)

Use Theorem 9.11 to determine the convergence or divergence of the p-series.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$$

1. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{1/5}}$
2. Divergent p-series with $p = \frac{1}{5} < 1$

Question : 10 (Comparision test)

Use the Direct Comparison Test to determine the convergence or divergence of the series.

$$\sum_{n=0}^{\infty} e^{-n^2}$$

1. $0 < \frac{1}{e^{n^2}} \leq \frac{1}{e^n}$
2. Therefore,

   $$\sum_{n=0}^{\infty} \frac{1}{e^{n^2}}$$

   converges by comparison with the convergent geometric series

   $$\sum_{n=0}^{\infty} \left(\frac{1}{e}\right)^n.$$

Question 11 (Limit comparision test)

Use the Limit Comparison Test to determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2+1}}$$

1. $\lim_{n\to\infty} \frac{1/\left(n\sqrt{n^2+1}\right)}{1/n^2} = \lim_{n\to\infty} \frac{n^2}{n\sqrt{n^2+1}}$
2. $= 1$
3. Therefore,

   $$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2+1}}$$

   converges by a limit comparison with the convergent p-series

   $$\sum_{n=1}^{\infty} \frac{1}{n^2}.$$

Question 12 (Ratio Test)

Use the Ratio Test to determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \frac{2^n}{n^2}$$

1. $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{2^{n+1}}{(n+1)^2} \cdot \frac{n^2}{2^n}\right|$
2. $= \lim_{n\to\infty} \frac{2n^2}{(n+1)^2}$
3. $= 2$
4. Therefore, by the Ratio Test, the series diverges.

Question 13 (Ratio test)

Determine the convergence or divergence of the series using any appropriate test from this chapter. Identify the test used.

$$\sum_{n=1}^{\infty} \frac{n7^n}{n!}$$

1. $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(n+1)7^{n+1}}{(n+1)!} \cdot \frac{n!}{n7^n}\right|$
2. $= \lim_{n\to\infty} \frac{7}{n}$
3. $= 0$
4. Therefore, by the Ratio Test, the series converges.

Question 14 (Alternating series Test)

Determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1}$$

1. $a_{n+1} = \frac{1}{2(n+1)-1}$
2. $< \frac{1}{2n-1}$
3. $= a_n$
4. $\lim_{n\to\infty} \frac{1}{2n-1} = 0$
5. Converges

Question 15 (Alternating series Test)

Determine the convergence or divergence of the series.

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$$

1. $a_{n+1} = \frac{1}{(n+1)!}$
2. $< \frac{1}{n!}$
3. $= a_n$
4. $\lim_{n\to\infty} \frac{1}{n!} = 0$
5. Converges by Theorem 9.14