

MATH152 CALCULUS II TUTORIAL – I

(16.02.2018)

Question 1 : (sequences)

Find the limit (if possible) of the sequence.

$$a_n = \frac{2n}{\sqrt{n^2 + 1}}$$

1. $\lim_{n \rightarrow \infty} \frac{2n}{\sqrt{n^2 + 1}} = \lim_{n \rightarrow \infty} \frac{2}{\sqrt{1 + (1/n^2)}}$
2. $= \frac{2}{1}$
3. $= 2$

Question 2 : (sequences)

Determine the convergence or divergence of the sequence with the given n th term. If the sequence converges, find its limit.

$$a_n = \frac{1 + (-1)^n}{n}$$

1. $\lim_{n \rightarrow \infty} \frac{1 + (-1)^n}{n} = 0$
2. Converges

Question 3 :

Determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2} \right)$$

1. $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2} \right) = \left(1 - \frac{1}{3} \right) + \left(\frac{1}{2} - \frac{1}{4} \right) + \left(\frac{1}{3} - \frac{1}{5} \right) + \left(\frac{1}{4} - \frac{1}{6} \right) + \dots$
2. $= 1 + \frac{1}{2}$
3. $= \frac{3}{2}$
4. Converges

Note: Find the form of s_n .

Question 4: (Geometric Series)

Verify that the infinite series converges.

$$\sum_{n=0}^{\infty} 2 \left(\frac{3}{4} \right)^n$$

1. Geometric series with $r = \frac{3}{4} < 1$
2. Converges by Theorem 9.6

Question 5: (Geometric Series)

Find the sum of the convergent series.

$$\sum_{n=0}^{\infty} \left(\frac{1}{2^n} - \frac{1}{3^n} \right)$$

1. $\sum_{n=0}^{\infty} \left(\frac{1}{2^n} - \frac{1}{3^n} \right) = \sum_{n=0}^{\infty} \left(\frac{1}{2} \right)^n - \sum_{n=0}^{\infty} \left(\frac{1}{3} \right)^n$
2. $= \frac{1}{1 - (1/2)} - \frac{1}{1 - (1/3)}$
3. $= 2 - \frac{3}{2}$
4. $= \frac{1}{2}$

Question 6:

Find the sum of the convergent series.

$$\sum_{n=0}^{\infty} \left(\frac{1}{2} \right)^n$$

1. $\sum_{n=0}^{\infty} \left(\frac{1}{2} \right)^n = \frac{1}{1 - (1/2)}$
2. $= 2$

Question 7: (n'th term test)

Determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \frac{3n-1}{2n+1}$$

1. $\lim_{n \rightarrow \infty} \frac{3n-1}{2n+1} = \frac{3}{2}$
2. $\neq 0$
3. Diverges

Question 8: (n'th term test)

Determine the convergence or divergence of the series.

$$\sum_{n=2}^{\infty} \frac{n}{\ln n}$$

1. Since $n > \ln(n)$, the terms $a_n = \frac{n}{\ln(n)}$ do not approach 0 as $n \rightarrow \infty$.
2. Hence, the series $\sum_{n=2}^{\infty} \frac{n}{\ln(n)}$ diverges.

Question 9: (p-test)

Use Theorem 9.11 to determine the convergence or divergence of the p-series.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$$

1. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{1/5}}$
2. Divergent p-series with $p = \frac{1}{5} < 1$

Question 10: (Comparison test)

Use the Direct Comparison Test to determine the convergence or divergence of the series.

$$\sum_{n=0}^{\infty} e^{-n^2}$$

1. $0 < \frac{1}{e^{n^2}} \leq \frac{1}{e^n}$
2. Therefore,

$$\sum_{n=0}^{\infty} \frac{1}{e^{n^2}}$$

converges by comparison with the convergent geometric series

$$\sum_{n=0}^{\infty} \left(\frac{1}{e}\right)^n$$

Question 11: (Limit comparison test)

Use the Limit Comparison Test to determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2+1}}$$

1. $\lim_{n \rightarrow \infty} \frac{1/(n\sqrt{n^2+1})}{1/n^2} = \lim_{n \rightarrow \infty} \frac{n^2}{n\sqrt{n^2+1}}$
2. $= 1$

3. Therefore,

$$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2+1}}$$

converges by a limit comparison with the convergent p-series

$$\sum_{n=1}^{\infty} \frac{1}{n^2}$$

Question 12 (Ratio Test)

Use the Ratio Test to determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \frac{2^n}{n^2}$$

1. $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{2^{n+1}}{(n+1)^2} \cdot \frac{n^2}{2^n} \right|$
2. $= \lim_{n \rightarrow \infty} \frac{2n^2}{(n+1)^2}$
3. $= 2$
4. Therefore, by the Ratio Test, the series diverges.

Question 13 (Ratio test)

Determine the convergence or divergence of the series using any appropriate test from this chapter. Identify the test used.

$$\sum_{n=1}^{\infty} \frac{n7^n}{n!}$$

1. $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(n+1)7^{n+1}}{(n+1)!} \cdot \frac{n!}{n7^n} \right|$
2. $= \lim_{n \rightarrow \infty} \frac{7}{n}$
3. $= 0$
4. Therefore, by the Ratio Test, the series converges.

Question 14 (Alternating series Test)

Determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1}$$

1. $a_{n+1} = \frac{1}{2(n+1)-1}$
2. $< \frac{1}{2n-1}$
3. $= a_n$
4. $\lim_{n \rightarrow \infty} \frac{1}{2n-1} = 0$
5. Converges

Question 15 (Alternating series Test)

Determine the convergence or divergence of the series.

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$$

1. $a_{n+1} = \frac{1}{(n+1)!}$
2. $< \frac{1}{n!}$
3. $= a_n$
4. $\lim_{n \rightarrow \infty} \frac{1}{n!} = 0$
5. Converges by Theorem 9.14

Question 16 (Alternating series Test)

Determine the convergence or divergence of the series.

$$\sum_{n=4}^{\infty} \frac{(-1)^n n}{n-3}$$

1. Diverges by the n th-Term Test.