Faculty of Arts and Sciences
Department of Mathematics
MATH152 Midterm II

Std. No: **Name :** **Surname :** **21.12.2018**

1	2	3	4	5	6	TOTAL

Duration : 90 mins

Q-1) Find the equations of tangent plane and normal line to the surface $x + x\cos z - y\sin z + y = 0$ at the point $P(2,-4,0)$.

$$\left.\begin{aligned} f_x &= 1+\cos z \\ f_y &= -\sin z + 1 \\ f_z &= -x\sin z - y\cos z \end{aligned}\right\} \Rightarrow \left.\begin{aligned} f_x(2,-4,0) &= 2 \\ f_y(2,-4,0) &= 1 \\ f_z(2,-4,0) &= 4 \end{aligned}\right\} \Rightarrow n = \langle 2,1,4 \rangle$$

<u>Equation of tangent plane :</u>

$$2(x-2) + (y+4) + 4(z-0) = 0$$
$$2x + y + 4z = 0$$

<u>Equation of the normal line</u>

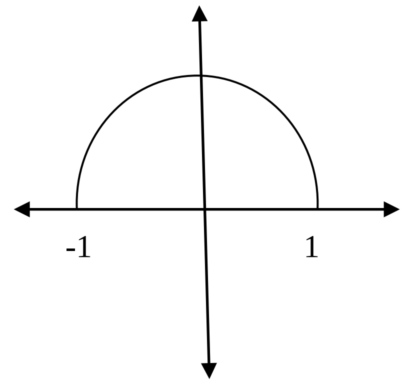
$$x = 2 + 2t$$
$$y = -4 + t$$
$$z = 4t$$
$$-\infty < t < \infty$$

Q-2) Evaluate $\iint_R (x^2+y^2)^{\frac{5}{2}}\, dA$ if R is the regoin bounded by the semi circle $y=\sqrt{1-x^2}$ and $x-$ axis.

$$\int_0^{\pi}\int_0^{1} r^6\, dr\, d\theta = \int_0^{\pi} \left.\frac{r^7}{7}\right|_0^1 d\theta = \int_0^{\pi} \frac{1}{7}\, d\theta = \frac{1}{7}\pi.$$

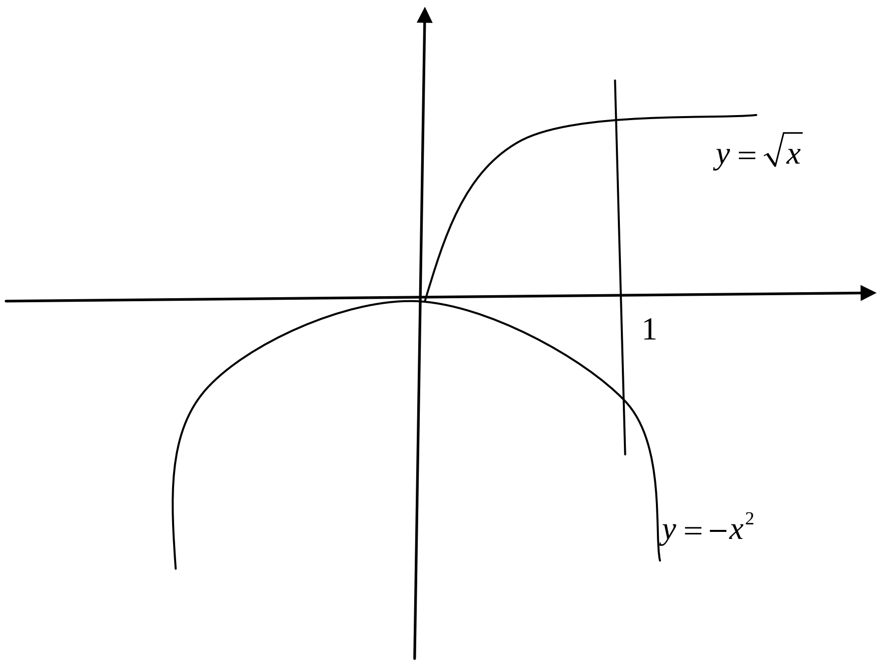
$0 \le \theta \le \pi$

$0 \le r \le 1$

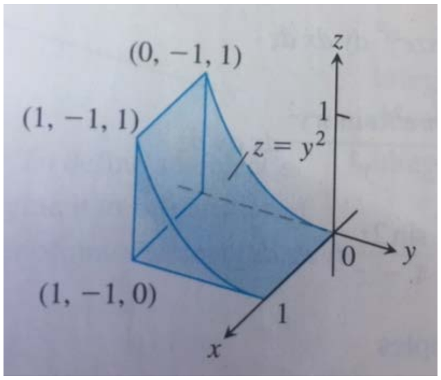


Q-3) (20 Points) Evaluate the area of the region bounded by the graphs of the equations
$y = -x^2$, $y = \sqrt{x}$ and $x = 1$.

$$\int_0^1 \int_{-x^2}^{\sqrt{x}} 1\,dy\,dx = \int_0^1 (\sqrt{x} + x^2)\,dx = \frac{2}{3}x^{\frac{3}{2}} + \frac{x^3}{3}\Bigg|_0^1 = \frac{2}{3} + \frac{1}{3} = 1$$



Q-4) Use a triple integral to find the volume of the region bounded by, $z = y^2$, $z = 0$, $x = 0$, $x = 1$, $y = 0$ and $y = -1$. (see the figure)



$$\left.\begin{array}{l} 0 \le x \le 1 \\ -1 \le y \le 0 \\ 0 \le z \le y^2 \end{array}\right\} \Rightarrow \int_0^1 \int_{-1}^0 \int_0^{y^2} 1\, dz\, dy\, dx = \int_0^1 \int_{-1}^0 y^2 dy\, dx = \int_0^1 \left. \frac{y^3}{3} \right|_{-1}^{0} dx = \frac{1}{3}$$

Q.5) (20 Points) Find extrema and saddle points of $f(x,y)=2x^3+2y^3-9x^2+3y^2-12y$

$$f_x(x,y)=6x^2-18x=6x(x-3)=0 \;\Rightarrow x=0\,and\;x=3$$
$$f_y(x,y)=6y^2+6y-12=0 \Rightarrow y=1and\;y=-2$$

Critical points : $(0,1),(0,-2),(3,1)\,and\,(3,-2)$

$$\left.\begin{array}{r} f_{xx}(x,y)=12x-18 \\ f_{yy}(x,y)=12y+6 \\ f_{xy}(x,y)=0 \end{array}\right\} \Rightarrow D(x,y)=(12x-18)\left(12y+6\right)=36(2x-3)(2y+1)$$

Critical Points	The value of D(x,y)	The value of f_{xx}	Conclusin
$(0,1)$	$D(0,1)<0$	No need	Saddle point
$(0,-2)$	$D(0,-2)>0$	$f_{xx}(0,-2)=-18<0$	Local Maximum
$(3,1)$	$D(3,1)>0$	$f_{xx}(3,1)=18>0$	Local Minimum
$(3,-2)$	$D(3,-2)<0$	No need	Saddle Point

Q-6) (20 Points) Use cylindrical coordinates to evaluate $\iiint_Q x^2\, dV$ where Q is the region bounded by the graphs of the equations $z = x^2 + y^2$ and $z = 2 - x^2 - y^2$ (see the figure).

$$\int_0^{2\pi}\int_0^1\int_{r^2}^{2-r^2} r^3\cos^2\theta\, dz dr d\theta = \int_0^{2\pi}\int_0^1 z\Big|_{r^2}^{2-r^2} r^3\cos^2\theta dr d\theta = \int_0^{2\pi}\int_0^1 (2-2r^2)r^3\cos^2\theta dr d\theta$$

$$= \int_0^{2\pi}\int_0^1 (2r^3 - 2r^5)\cos^2\theta\ dr d\theta = \int_0^{2\pi} (\frac{r^4}{2} - \frac{r^6}{3})\Big|_0^1 \cos^2\theta\ dr d\theta = \int_0^{2\pi} \frac{1}{6}\cos^2\theta\ d\theta$$

$$= \int_0^{2\pi} \frac{1}{12}(1+\cos 2\theta)\ d\theta = \frac{1}{12}(\theta + \frac{\sin 2\theta}{2})\Big|_0^{2\pi} = \frac{\pi}{6}.$$