MATH 152 MIDTERM II EXAM SOLUTION

FACULTY OF ARTS AND SCIENCES

DEPARTMENT OF MATHEMATICS
EMU
2015-2016 Fall

Faculty of Arts and Sciences
Department of Mathematics
MATH152
Midterm II

Std. No: **Name :** **Surname :** **19.12.2015**

1	2	3	4	5	6	TOTAL

Duration : 90 mins

Q-1) Investigate the following limits

a) (10 Points) $\lim_{(x,y)\to(0,0)} \frac{x^3 y}{2y^4+3x^4}$

$$y = mx \Rightarrow \lim_{\substack{x\to 0\\ y=mx}} \frac{x^4 m}{2x^4m^4+3x^4} = \frac{m}{2m^4+3} \text{ depends on m} \Rightarrow \text{limit does not exist}$$

b) (10 Points) $\lim_{(x,y)\to(-1,2)} \frac{x^2+2x+1}{(x+1)^2(y+1)}$

$$= \lim_{\substack{x\to -1\\ y\to 2}} \frac{(x+1)^2}{(x+1)^2(y+1)} = \lim_{\substack{x\to -1\\ y\to 2}} \frac{1}{y+1} = \frac{1}{3}$$

Q-2) **a)** (10 Points) Use chain rule to find $\frac{\partial w}{\partial x}$ if $w = u^2 + 4vs$ with $u = x^3, v = x^2y^3$ and $s = e^{4y}$

$$\frac{\partial w}{\partial x} = \frac{\partial w}{\partial u}\frac{du}{dx} + \frac{\partial w}{\partial v}\frac{\partial v}{\partial x}$$

$$= 2u3x^2 + 4s2xy^3 = 6x^2x^3 + 8xy^3e^{4y}$$

$$= 6x^5 + 8xy^3e^{4y}$$

b) (10 Points) Find the equation of tangent plane to the surface $xy - z = 6$ at the point $P(-2,-3,6)$.

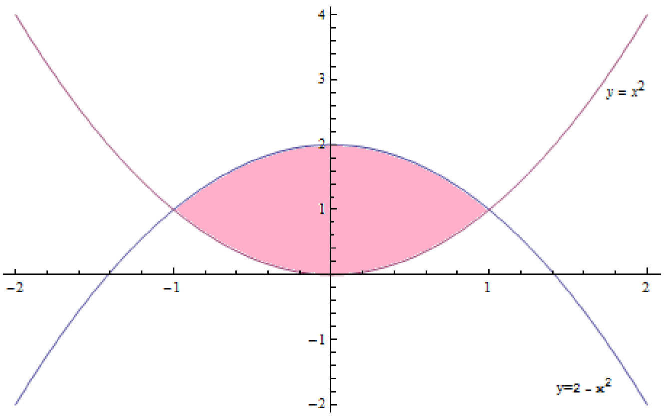
$$f(x,y,z) = xy - z - 6 = 0$$

$$\left.\begin{array}{l} f_x = y \\ f_y = x \\ f_z = -1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} f_x(-2,-3,6) = -3 \\ f_y(-2,-3,6) = -2 \\ f_z(-2,-3,6) = -1 \end{array}\right\} \Rightarrow \text{Equation of the tangent plane} \Rightarrow$$

$$-3(x+2) - 2(y+3) - (z-6) = 0$$

$$3x + 2y + z = -6$$

Q-3) Use double integral to find the area of the region bounded by the graphs of the equations $y = 2 - x^2$ and $y = x^2$



$$\left.\begin{array}{l} -1 \le x \le 1 \\ x^2 \le y \le 2 - x^2 \end{array}\right\} \Rightarrow \text{Area} = \iint_R 1dA = \int_{-1}^{1}\int_{x^2}^{2-x^2} dydx = \int_{-1}^{1} y\Big|_{x^2}^{2-x^2} dx = \int_{-1}^{1}\left(\left(2-x^2\right) - x^2\right)dx$$

$$= \int_{-1}^{1}\left(2 - 2x^2\right)dx = 2x - \frac{2}{3}x^3\Big|_{-1}^{1} = \left(2 - \frac{2}{3}\right) - \left(-2 + \frac{2}{3}\right) = \frac{8}{3}$$

Q-4) Let $f(x,y,z)=x^3y^2z$ then find

a) (14 Points) Directional derivative of f at the point $P(1,-1,2)$ in the direction of $u=\langle 2,1,3\rangle$

$$\|\vec{u}\|=\sqrt{4+1+9}=\sqrt{14}$$

$$\frac{\vec{u}}{\|\vec{u}\|}=\left\langle \frac{2}{\sqrt{14}},\frac{1}{\sqrt{14}},\frac{3}{\sqrt{14}}\right\rangle,\quad \vec{\nabla} f=\left\langle f_x,f_y,f_z\right\rangle=\left\langle 3x^2y^2z,2x^3yz,x^3y^2\right\rangle\Rightarrow \vec{\nabla} f\Big|_{P(1,-1,2)}=\langle 6,-4,1\rangle$$

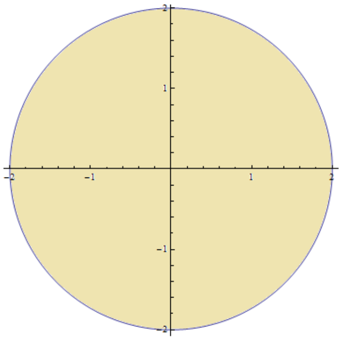
$$D_u f\Big|_{P(1,-1,2)}=\left\langle \frac{2}{\sqrt{14}},\frac{1}{\sqrt{14}},\frac{3}{\sqrt{14}}\right\rangle\cdot\langle 6,-4,1\rangle=\frac{12-4+3}{\sqrt{14}}=\frac{11}{\sqrt{14}}$$

b) (6 Points) In which direction f has maximal directional derivative at $P(1,-1,2)$ and find the value of maximal directional derivative of f in that direction.

$\vec{\nabla} f(1,-1,2)=6\vec{i}-4\vec{j}+\vec{k}$ direction of max. directional derivative

$\left\|\vec{\nabla} f(1,-1,2)\right\|=\sqrt{36+16+1}=\sqrt{53}$ is the maximum directional derivative

Q-5) Use polar coordinates to evaluate $\iint\limits_R \frac{1}{\sqrt{x^2+y^2}}dA$ where R is the region bounded by the circle $x^2+y^2=4$.



$x=r\cos\theta$

$y=r\sin\theta$

$$R=\{(r,\theta):0\le r\le 2,\quad 0\le\theta\le 2\pi\}\Rightarrow \iint\limits_R \frac{1}{\sqrt{x^2+y^2}}dA=\int_0^{2\pi}\int_0^2 \frac{1}{r}\,r\,dr\,d\theta=4\pi$$

Q.6) Find extrema and saddle points of $f(x,y)=\frac{1}{3}x^3+\frac{1}{3}y^3-\frac{5}{2}x^2+4x-4y+6$

$f_x = x^2-5x+4=0 \Rightarrow x_1=4, x_2=1$

$f_y = y^2-4=0 \Rightarrow y_1=2, y_2=-2$

Critical points are $(4,2),(4-2),(1,2),(1,-2)$

$$\left.\begin{array}{l} f_{xx}=2x-5 \\ f_{yy}=2y \\ f_{xy}=0 \end{array}\right\} \Rightarrow \Delta = 2y(2x-5)$$

Critical Points	The value of Δ	The value of f_{xx}	Classification
(4,2)	$\Delta = 12 > 0$	$f_{xx}=2(4)-5=3>0$	Local minimum
(4,-2)	$\Delta = -12 < 0$	Irrelevant	Saddle
(1,2)	$\Delta = -12 < 0$	Irrelevant	Saddle
(1,-2)	$\Delta = 12 > 0$	$f_{xx}=2(-2)-5=-9<0$	Local maximum