CALCULUS II MIDTERM II EXAM SOLUTION

Eastern Mediterranean University

Department Of Mathematics

EMU

2016-2017 SPRING]

Faculty of Arts and Sciences
Department of Mathematics
MATH152 Midterm II

Std. No: **Name :** **Surname :** **10.05.2017**

1	2	3	4	5	6	TOTAL

Duration : 90 mins

Q-1) Investigate the following limit

a) (10 Points) $\lim\limits_{(x,y)\to(0,0)} \dfrac{3x^3y}{6y^2x+9x^5}$

$y = mx^2 \Rightarrow \quad \lim\limits_{(x,y)\to(0,0)} \dfrac{3x^3mx^2}{6\left(mx^2\right)^2 x+9x^5} = \lim\limits_{(x,y)\to(0,0)} \dfrac{3mx^5}{6m^2x^5+9x^5}$

$= \lim\limits_{(x,y)\to(0,0)} \dfrac{3mx^5}{x^5\left(6m^2+9\right)} = \dfrac{3m}{6m^2+9}$ on parabola $y = x^2\ (m=1)$ limit is $\dfrac{1}{5}$,

on parabola $y = 2x^2\ (m=2)$ limit is $\dfrac{2}{11}$ from two path rule limit Does not exist.

b) (10 Points) Find the domain of $f(x,y) = \dfrac{xy}{\sqrt{x-2}} + \ln\left(y-2\right)$

$x-2>0 \ and\ \ y-2>0 \ \Rightarrow\ x>2 \ and\ \ y>2.$

$Domain\ f = \left\{\left(x,y\right): x>2\ and\ y>2\right\}$

Q-2) (20 Points) Find equations for the tangent plane and the normal line to the graph of $\sqrt{x}+\sqrt{y}-z=0$ at the point $P(1,4,3)$.

$$F(x,y,z)=\sqrt{x}+\sqrt{y}-z \Rightarrow \overrightarrow{\nabla F}=\left\langle \frac{1}{2\sqrt{x}},\frac{1}{2\sqrt{y}},-1\right\rangle \Rightarrow \overrightarrow{\nabla F}(1,4,3)=\left\langle \frac{1}{2},\frac{1}{4},-1\right\rangle$$

Tangent plane: $\frac{1}{2}(x-1)+\frac{1}{4}(y-4)-(z-3)=0$, $\Rightarrow 2x+y-4z+6=0$

Normal Line : $x=1+\frac{1}{2}t,\ \ y=4+\frac{1}{4}t,\ \ z=3-t,\ -\infty<t<\infty$

Q-3) (20 Points) Find local extrema and saddle points of
$f(x,y)=x^4-2x^2+2y^2-8y+11$

$$\left.\begin{array}{l} f_x=4x^3-4x=0 \\ f_y=4y-8=0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 4x\left(x^2-1\right)=0 \Rightarrow x=0, x=1 \, and \, x=-1 \\ y=2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \text{critical ponits are} \\ (0,2),\ (1,2)\, and \,(-1,2) \end{array}\right\}$$

$$\mathrm{D}(x,y)= f_{xx}f_{yy}-\left(f_{xy}\right)^2=\left(12x^2-4\right)4-0=16\left(3x^2-1\right)$$

Critical point (x,y)	$D(x,y)$	f_{xx}	Conclusion
$(0,2)$	$D(0,2)=-16$	Irrelavant	Saddle point
$(1,2)$	$D(1,2)=32>0$	$f_{xx}(1,2)=8$	Local minimum
$(-1,2)$	$D(-1,2)=32>0$	$f_{xx}(-1,2)=8$	Local minimum

Q-4) **a) (10 Points)** Use <u>chain rule</u> to find $\frac{\partial w}{\partial s}$ if $w = xe^y - yz$ with $x = -s + t^2$, $y = (t+s)^2$ and $z = \ln(t+s)$. (No points will be given to direct computation)

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial s}$$

$$= -e^y + (xe^y - z)2(t+s) - y\frac{1}{t+s}$$

$$= -e^{(t+s)^2} + \left((-s+t^2)e^{(t+s)^2} - \ln(t+s)\right)2(t+s) - (t+s)^2\frac{1}{t+s}$$

$$= -e^{(t+s)^2} + \left((-s+t^2)e^{(t+s)^2} - \ln(t+s)\right)2(t+s) - (t+s)$$

b) (10 Points) Suppose that $F(x,y) = xe^y + \sin(xy) + y = 0$ defines y as a differentiable function of x. Then use partial derivatives of $F(x,y)$ to find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{e^y + y\cos(xy)}{xe^y + x\cos(xy) + 1}$$

Q-5) (20 Points) a) Use polar coordinates to evaluate $\iint\limits_R \frac{1}{x^2+y^2+1}dA$ where the region R is the upper semi-disc bounded by the circle $x^2+y^2=4$ and $x-$axis.

$$\int_0^{\pi}\int_0^{2}\frac{1}{\left(r^2+1\right)}rdrd\theta=\frac{1}{2}\int_0^{\pi}\int_1^{5}\frac{du}{u}d\theta=\frac{1}{2}\int_0^{\pi}\ln|u|\Big|_1^5\,d\theta=\frac{1}{2}\ln 5\theta\Big|_0^{\pi}=\frac{\pi}{2}\ln 5$$

b) Rewrite the double integral $\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} xy\,dydx$ in polar coordinates (Do not evaluate the integral).

$$\begin{cases} y=\sqrt{1-x^2} \\ y^2+x^2=1 \\ 0\le r\le 1 \\ 0\le\theta\le 2\pi \end{cases}$$

$$\Rightarrow\int_0^{2\pi}\int_0^{1} r\cos\theta r\sin\theta rdrd\theta$$

$$=\int_0^{2\pi}\int_0^{1}\cos\theta\sin\theta r^3drd\theta$$

Q.6) (20 Points) Use double integral to find the area of the region bounded by the graphs of the equations $y=x^2$, $y=3x$.

$$x^2=3x\Rightarrow x^2-3x=0\Rightarrow x=0, x=3$$

$$\begin{cases} 0\le x\le 3 \\ x^2\le y\le 3x \end{cases}\Rightarrow A=\int_0^{3}\int_{x^2}^{3x}1dydx=\int_0^{3}\left(3x-x^2\right)dx=\frac{3x^2}{2}-\frac{x^3}{3}\Big|_0^3=\frac{9}{2}$$