Eastern Mediterranean University

Calculus II

MT-1 Examination Solution

Department of Mathematics

2018-19

SPRING

Faculty of Arts and Sciences
Department of Mathematics
MATH152, Midterm I **11.04.2019**

Std.: **Name –Surname :** **Gr:**

1	2	3	4	5	6	TOTAL

Duration : 90 mins

Q-1) (20 points) Find the center, radius and interval of convergence for $\sum_{n=1}^{\infty}\frac{(2x+5)^n}{3^n}$

$$a_n=\frac{(2x+5)^n}{3^n},\ a_{n+1}=\frac{(2x+5)^{n+1}}{3^{n+1}}$$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\left|\frac{(2x+5)^{n+1}}{3^{n+1}}\frac{3^n}{(2x+5)^n}\right|=\frac{|2x+5|}{3}$$

1) $\frac{|2x+5|}{3}<1\Rightarrow\ -4<x<-1$ absolutely convergent $\Rightarrow$ convergent

2) $x<-4\ or\ x>-1$ divergent

3) $x=-4\Rightarrow\sum_{n=1}^{\infty}\frac{(2x+5)^n}{3^n}=\sum_{n=1}^{\infty}(-1)^n\Rightarrow$ divergent (alternating series test).

$x=-1\Rightarrow\sum_{n=1}^{\infty}\frac{(2x+5)^n}{3^n}=\sum_{n=1}^{\infty}1\Rightarrow$ divergent (n^{th} term test)

Center of convergence : $x=-\frac{5}{2}$,

Interval of convergence : $(-4,-1)$,

Redius of convergence : $R=\frac{3}{2}$.

Q.2) (10 points) a) Let C be the curve which is represented by the vector valued function $\overrightarrow{r(t)} = \frac{1}{3}t^3 i + \frac{2}{5}t^{5/2} j + \frac{1}{4}t^2 k,\ \ 0 \le t \le 2.$ Find the arc length of C.

$$\frac{dx}{dt} = t^2, \quad \frac{dy}{dt} = \sqrt{2}\ t^{\frac{3}{2}}, \quad \frac{dz}{dt} = t, \ \Rightarrow arc-length = \int_0^2 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$$

$$= \int_0^2 \sqrt{\left(t^2\right)^2 + \left(\sqrt{2}t^{\frac{3}{2}}\right)^2 + \left(t\right)^2}\, dt = \int_0^2 \sqrt{t^4 + 2t^3 + t^2}\, dt = \int_0^2 \sqrt{(t^2+t)^2}\, dt = \int_0^2 (t^2+t)dt = \left.\frac{t^3}{3} + \frac{t^2}{2}\right|_0^2 = \frac{14}{3}.$$

b) (10 points) Let C be the the curve with parametric equation $x = 2t^2$ and $y = 8t^3 + t$, then find the equation of the tangent line to the curve C corresponding to the point $t = 1$.

$$\frac{dx}{dt} = 4t, \ \frac{dy}{dt} = 24t^2 + 4 \Rightarrow m = \left.\frac{dy}{dx}\right|_{t=1} = \left.\frac{24t^2+4}{4t}\right|_{t=1} = 7,$$

$t = 1 \Rightarrow \left(x, y\right) = (2,12)$ The equation of the tangent line is $y - 12 = 7(\mathrm{x} - 2) \Rightarrow \mathrm{y} = 7x - 2.$

<u>II. Way</u>

$$\frac{dx}{dt} = 4t, \ \frac{dy}{dt} = 24t^2 + 4 \Rightarrow \left.\frac{dx}{dt}\right|_{t=1} = 4, \quad \left.\frac{dy}{dt}\right|_{t=1} = 28.$$

The direction vector, of the tangent line is $\vec{n} = \langle 4, 28 \rangle$

The parametric equation of the tangent line is; $\begin{array}{l} x = 2 + 4t \\ y = 12 + 28t \\ -\infty < t < \infty. \end{array}$

Q-3) (10 points) a) Determine whether the line $l: x = 3 + 2t,\ \ y = 2 + 2t,\ \ z = 3 + 4t,$ $-\infty < t < \infty,$ lies on the plane $x + y - z = 2$.

$(3+2t)+(2+2t)-(3+4t)=2 \Rightarrow$ The line is on the plane.

b) (10 points) Find the equation of the plane, that contains the points $P(3,1,2),\ \ Q(-1,2,-3)$ and $R(1,0,3)$.

$$\overrightarrow{PQ} = \langle -4,1,-5 \rangle,\ \ \overrightarrow{PR} = \langle -2,-1,1 \rangle \Rightarrow \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -4 & 1 & -5 \\ -2 & -1 & 1 \end{vmatrix} = \langle -4,14,6 \rangle$$

$$-4(x-3)+14(\mathrm{y}-1)+6(\mathrm{z}-2)=0 \ \ \Rightarrow \ \ -4x+14\mathrm{y}+6\mathrm{z}=14 \ \ \Rightarrow \ \ -2x+7\mathrm{y}+3\mathrm{z}=7.$$

Q-4) Given $\vec{u} = \langle 3,-2,-1 \rangle$ and $\vec{v} = \langle 2,3,-3 \rangle$ then,

a) (10 points) Find the cross product of $2\vec{u}$ and $-\vec{v}$.

$\overrightarrow{2u} = \langle 6,-4,-2 \rangle$, $\overrightarrow{-v} = \langle -2,-3,3 \rangle$

$$(\overrightarrow{2u})x(\overrightarrow{-v}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & -4 & -2 \\ -2 & 3 & 3 \end{vmatrix} = \langle -18,-14,-26 \rangle$$

b) (10 points) the vector projection of $2\vec{u}$ onto $-\vec{v}$.

$$\Pr oj_{-\vec{v}}(\overrightarrow{2u}) = \frac{\langle 6,-4,-2 \rangle \langle -2,-3,3 \rangle}{\left\| \langle -2,-3,3 \rangle \right\|^2} = \frac{-6}{22} \langle -2,-3,3 \rangle = \left\langle \frac{6}{11}, \frac{9}{11}, \frac{-9}{11} \right\rangle.$$

Q-5) a) (10 points) Find all first order partial derivatives of

$$f(x,y,z) = x^2 y + y^2 e^{xy} + \cos(zx).$$

$$f_x(x,y,z) = 2xy + y^3 e^{xy} - z\sin(zx).$$

$$f_y(x,y,z) = x^2 + xy^2 e^{xy} + 2ye^{xy}.$$

$$f_z(x,y,z) = -x\sin(zx).$$

b) (10 points) Use **Chain rule** to evaluate $\frac{\partial w}{\partial y}$ where $w = s^2 t + t^2 s$, $t = x^2 + 3y$ and $s = yx^2$.

$$\frac{\partial w}{\partial y} = \frac{\partial w}{\partial s}\frac{\partial s}{\partial y} + \frac{\partial w}{\partial t}\frac{\partial t}{\partial y} = \left(2st + t^2\right)x^2 + 3\left(s^2 + 2st\right) = 2yx^6 + 9y^2x^4 + x^6 + 12x^4y + 27x^2y^2.$$

Q-6) a) (10 points) Investigate the following limit

$$\lim_{(x,y)\to(0,0)} \frac{x^2 y + 2x^4}{2x^4 + y^2}.$$

On Paths $y = mx^2 \Rightarrow \lim_{(x,y)\to(0,0)} \frac{mx^4 + 2x^4}{2x^4 + m^2x^4} = \frac{m+2}{2+m^2}$

On On $y = 0 \Rightarrow \text{limit} = 1$, On $x = 0 \Rightarrow \text{limit} = 0$,

By two path rule limit does not exists.

b) (10 points) Find the power series representation of the function $f(x) = \frac{x^2}{2-4x}$ in powers of x .

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^n + \cdots = \sum_{n=0}^{\infty} x^n, \quad -1 < x < 1.$$

$$\frac{1}{2-4x} = \frac{1}{2}\left(1 + 2x + 4x^2 + 8x^3 + \cdots + 2^n x^n + \cdots\right) = \sum_{n=0}^{\infty} 2^{n-1} x^n, \quad -\frac{1}{2} < x < \frac{1}{2}.$$

$$\frac{x^2}{2-4x} = x^2\left(\frac{1}{2} + x + 2x^2 + 4x^3 + \cdots + 2^{n-1} x^n + \cdots\right) = \sum_{n=0}^{\infty} 2^{n-1} x^{n+2}, \quad -\frac{1}{2} < x < \frac{1}{2}.$$