MATH 152 MIDTERM EXAM SOLUTION (MT-1)

SPRING 2016-2017

EASTERN MEDITERRANEAN UNIVERSITY

FACULTY OF ARTS AND SCIENCES
DEPARTMENT OF MATHEMATICS

Faculty of Arts and Sciences
Department of Mathematics
MATH152, Midterm I

Std.: **Name –Surname** **Gr:** **09.04.2017**

1	2	3	4	5	6	TOTAL

Duration : 90 mins **Signature:**

Q-1) (20 points) Find the center, radius and interval of convergence for $\sum_{n=1}^{\infty} \frac{(x-4)^n}{(n+1)^2}$

Solution:

$$a_n = \frac{(x-4)^n}{(n+1)^2}, \quad a_{n+1} = \frac{(x-4)^{n+1}}{(n+2)^2}$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(x-4)^{n+1}}{(n+2)^2}\frac{(n+1)^2}{(x-4)^n}\right| = \lim_{n\to\infty}\left|(x-4)\left(\frac{n+1}{n+2}\right)^2\right| = |x-4|$$

if $|x-4|<1 \Rightarrow -1 < x-4 < 1 \Rightarrow 3 < x < 5 \rightarrow$ Absolutely convergent, therefore convergent.

if $x=3 \Rightarrow \sum_{n=1}^{\infty}\frac{(-1)^n}{(n+1)^2} \rightarrow$ Converges from alternating series test

if $x=5 \Rightarrow \sum_{n=1}^{\infty}\frac{1}{(n+1)^2} \rightarrow$ Converges from limit comparison test (use $b_n = \frac{1}{n^2}$).

Thus interval of convergence : $[3,5]$, center of convergence is $x=4$, radius of convergence: R=1.

Q-2) (20 points) Find the equation of the plane contaning following lines

$$l_1 : x = -1+t, y = 2+t, \ z = 1-t; \ -\infty < t < \infty$$
$$l_2 : x = 1-4s, y = 1+2s, \ z = 2-2s; -\infty < s < \infty.$$

Solution:

Direction vector of line 1 $\Rightarrow n_1 = i + j - k$

Direction vector of line 2 $\Rightarrow n_2 = -4i + 2j - 2k$

Normal vector of the plane: $n = n_1 \times n_2$.

$$n = \begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ -4 & 2 & -2 \end{vmatrix} = 6j + 6k$$

The point A(1,1,2) lies on the plane so the equation of the plane is y + z = 3.

Q-3) (20 points) Use the following power series representation

$$\frac{1}{1+x} = 1 - x + x^2 + \cdots + (-1)^n x^n + \cdots, \ (-1 < x < 1)$$

a) to find power series representations of $f(x) = \frac{x}{2}\ln(1+x)$ in powers of x.

b) use $P_3(x)$ to find the approximate value of $\ln(0.5)$.

Solution:

a) $$\frac{1}{1+x} = 1 - x + x^2 + \cdots + (-1)^n x^n + \cdots = \sum_{n=0}^{\infty} (-1)^n x^n, \ -1 < x < 1.$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^n \frac{x^{n+1}}{n+1} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}, \quad -1 < x < 1.$$

$$\frac{x}{2}\ln(1+x) = \frac{x^2}{2} - \frac{x^3}{4} + \frac{x^4}{6} - \frac{x^5}{8} + \cdots + (-1)^n \frac{x^{n+2}}{2(n+1)} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+2}}{2(n+1)}, \ -1 < x < 1.$$

b) 1^{st} *way* : (By using the Maclaurin series of $\ln(1+x)$.

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^n \frac{x^{n+1}}{n+1} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}, \quad -1 < x < 1.$$

$$P_3(x) = x - \frac{x^2}{2} + \frac{x^3}{3} \ \text{and} \ \ln(0.5) \approx -\frac{1}{2} - \frac{1}{8} - \frac{1}{24} = -\frac{2}{3}.$$

2^{nd} *way* : (By using the Maclaurin series of $\frac{x}{2}\ln(1+x)$.

$$\frac{x}{2}\ln(1+x) = \frac{x^2}{2} - \frac{x^3}{4} + \frac{x^4}{6} - \frac{x^5}{8} + \cdots + (-1)^n \frac{x^{n+2}}{2(n+1)} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+2}}{2(n+1)}, \quad -1 < x < 1.$$

$$\frac{-1}{4} P_3(x) = \frac{x^2}{2} - \frac{x^3}{4} \ \text{and} \quad \ln(0.5) \approx -4\left(\frac{1}{8} - \frac{1}{32}\right) = -\frac{5}{8}.$$

Q.4) (20 points) Given $\vec{u} = i + 2j - 4k$, $\vec{v} = -3i + 2j - k$ and $\vec{w} = 2i + 2j + k$ then find

a) $\dfrac{2\vec{u} - 3\vec{v}}{\|\vec{w}\|}$.

$$\frac{2\vec{u} - 3\vec{v}}{\|\vec{w}\|} = \frac{2\langle 1,2,-4\rangle - 3\langle -3,2,-1\rangle}{\sqrt{9}} = \left\langle \frac{11}{3}, -\frac{2}{3}, -\frac{5}{3} \right\rangle$$

b) the projection of $2\vec{u} - 3\vec{v}$ onto $\vec{w}$.

$$proj_{\vec{w}}\left(2\vec{u} - 3\vec{v}\right) = \frac{\left\langle \frac{11}{3}, -\frac{2}{3}, -\frac{5}{3} \right\rangle \langle 2,2,1\rangle}{9} \langle 2,2,1\rangle = \frac{13}{9}\langle 2,2,1\rangle = \left\langle \frac{26}{9}, \frac{26}{9}, \frac{13}{9} \right\rangle$$

Q.5) (20 points) Let C be the curve represented by the vector-valued function $r(t) = \cos\left(2t\right)\, i + 3tj + \sin\left(2t\right)k.$ Then

a) find $r'(t)$

b) find arc length of the part of the curve where $0 \le t \le \pi$.

c) find unit tangent vector to the curve C at $t = \dfrac{\pi}{2}$.

__**Solution:**__

a) $r'(t) = -2\sin\left(2t\right)\, i + 3j + 2\cos\left(2t\right)k.$

b) Arc length= $\int_0^{\pi} \sqrt{4\sin^2\left(2t\right) + 9 + 4\cos^2(2t)}dt = \int_0^{\pi} \sqrt{4+9}dt = \sqrt{13}\pi.$

c) $r'(t) = -2\sin\left(2t\right)\, i + 3j + 2\cos\left(2t\right)k,$

$$r'(\frac{\pi}{2}) = -2\sin\left(\pi\right)\, i + 3j + 2\cos\left(\pi\right)k$$

$$r'(\frac{\pi}{2}) = 3j - 2k,$$

Unit tangent vector is to the curve C at $t = \dfrac{\pi}{2}$ is:

$$\frac{r'(\frac{\pi}{2})}{\left\|r'(\frac{\pi}{2})\right\|} = \frac{\langle 0,3,-2\rangle}{\sqrt{13}} = \left\langle 0, \frac{3}{\sqrt{13}}, \frac{-2}{\sqrt{13}} \right\rangle$$

Q-6) (20 points) Determine whether two lines intersect each other and if so find the point of intersection.

$l_1 : x = 1 + 2t, y = 2 + t, \; z = 1 - 2t; \; -\infty < t < \infty$

$l_2 : x = 1 - 4s, y = 6 + 2s, \; z = -5 - 2s; \; -\infty < s < \infty.$

Solution:

$$\begin{array}{lcll} 1 - 4s = 1 + 2t, & & (1)\; 2t + 4s = 0, & & 2t + 4s = 0, \\ 2 + t = 6 + 2s & \Rightarrow & (2)\; t - 2s = 4 & \Rightarrow & \underline{-2t + 2s = -6} \\ 1 - 2t = -5 - 2s & & (3)\; -2t + 2s = -6 & & 6s = -6 \Rightarrow s = -1, \; t = 2 \end{array}$$

$\underline{check\; eqn\,(2)}$

$$\begin{array}{ll} 2 + t = 6 + 2s & \Rightarrow \\ 2 + 2 = 6 + 2(-1) & \\ 4 = 4 & \end{array}$$

Use $t = 2$ in l_1 or $s = -1$ in l_2,

$$\left. \begin{array}{l} x = 5 \\ y = 4 \\ z = -3 \end{array} \right\} \Rightarrow \text{The intersection point is } P(5, 4, -3)$$