MATH 152 MIDTERM EXAM SOLUTION

Fall 2015-2016

EASTERN MEDITERRANEAN UNIVERSITY

FACULTY OF ARTS AND SCIENCES
DEPARTMENT OF MATHEMATICS

Faculty of Arts and Sciences
Department of Mathematics
MATH152
Midterm I

Std.: **Name –Surname** **Gr:** **20.11.2015**

1	2	3	4	5	6	TOTAL

Duration : 90 mins **Signature:**

Q-1) (20 points) Find the interval of convergence for $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n}$

$$a_n = \frac{(x-3)^n}{n}, \quad a_{n+1} = \frac{(x-3)^{n+1}}{n+1}$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(x-3)^{n+1}}{n+1}\frac{n}{(x-3)^n}\right| = \lim_{n\to\infty}\left|(x-3)\frac{n}{n+1}\right| = |x-3|$$

$|x-3|<1 \Rightarrow -1 < x-3 < 1 \Rightarrow 2 < x < 4 \rightarrow$ Absolutely convergent

if $x=2 \Rightarrow \sum_{n=1}^{\infty}\frac{(-1)^n}{n} \rightarrow$ Converges from alternating series test

if $x=4 \Rightarrow \sum_{n=1}^{\infty}\frac{1}{n} \rightarrow$ Diverges from p- series test (Harmonic series) thus

interval of convergence is $[2,4)$

Q-2) a) (20 points) a) Find the parametric equation of the line through points $P(1,0,2)$ and $Q(3,-1,0)$.

Normal vector=$\overrightarrow{PQ} = Q - P = 2i - j - 2k$

A parametric equation of the line is

$x = 1 + 2t,\quad y = -t,\quad z = 2 - 2t,\quad -\infty < t < \infty$

b) Determine whether the line $x = 1 + t$, $y = -2 + 3t$, $z = -2 + 2t$ intersect the plane $2x + y + z = 2$. If so find the point of intersection.

$$2(1+t) + (-2+3t) + (-2+2t) = 2 \Rightarrow t = \frac{4}{7} \Rightarrow \text{line intersects the plane}$$

$$t = \frac{4}{7} \Rightarrow x = 1 + \frac{4}{7} = \frac{11}{7},\quad y = -2 + \frac{12}{7} = \frac{-2}{7},\quad z = -2 + \frac{8}{7} = \frac{-6}{7}$$

Intersection point is $\text{P}\left(\frac{11}{7}, \frac{-2}{7}, \frac{-6}{7}\right)$

Q-3) (20 points) Given $\vec{u} = \frac{1}{\sqrt{2}} i + \frac{1}{\sqrt{2}} k$ and $\vec{v} = \frac{1}{\sqrt{2}} i - \frac{1}{\sqrt{2}} j$, find

a) The angle between $\vec{u}$ and $\vec{v}$ and express it interms of π.

$$Cos\theta = \frac{u.v}{\|u\|\|v\|} = \frac{\frac{1}{2}}{\sqrt{\frac{1}{2} + \frac{1}{2}}\sqrt{\frac{1}{2} + \frac{1}{2}}} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

b) Unit vector in the direction of $2\vec{u} + \vec{v}$.

$$2\vec{u} + \vec{v} = \left(\frac{2}{\sqrt{2}}\vec{i} + \frac{2}{\sqrt{2}}\vec{k}\right) + \left(\frac{1}{\sqrt{2}}\vec{i} - \frac{1}{\sqrt{2}}\vec{j}\right) = \frac{3}{\sqrt{2}}\vec{i} - \frac{1}{\sqrt{2}}\vec{j} + \frac{2}{\sqrt{2}}\vec{k}$$

$$\|2\vec{u} + \vec{v}\| = \sqrt{\frac{9}{2} + \frac{1}{2} + \frac{4}{2}} = \sqrt{7} \Rightarrow \frac{2\vec{u} + \vec{v}}{\|2\vec{u} + \vec{v}\|} = \frac{3}{\sqrt{14}}\vec{i} - \frac{1}{\sqrt{14}}\vec{j} + \frac{2}{\sqrt{14}}\vec{k}$$

Q.4) (20 points) a) Find the equation of the plane through points $P(1,1,0)$, $Q(2,-1,1)$ and $R(3,1,2)$

$$\overrightarrow{PQ} = Q - P = \langle 1, -2, 1 \rangle$$
$$\overrightarrow{PR} = R - P = \langle 2, 0, 2 \rangle$$
$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & 1 \\ 2 & 0 & 2 \end{vmatrix} = -4\vec{i} + 4\vec{k}$$

Equation of the plane is

$$-4(x-1) + 4(z-0) = 0 \Rightarrow x - z = 1$$

b) Find the area of the triangle with vertices $P(1,1,0)$, $Q(2,-1,1)$ and $R(3,1,2)$

$$\text{Area} = \frac{\left\| \overrightarrow{PQ} \times \overrightarrow{PR} \right\|}{2} = \frac{\left\| \langle -4, 0, 4 \rangle \right\|}{2} = \sqrt{8}$$

Q.5) (20 points) a) Use the following power series representation

$$\frac{1}{1+x} = 1 - x + x^2 + \cdots + (-1)^n x^n + \cdots, \ (-1 < x < 1)$$

to find power series representation of $\ln(1+x)$ in powers of x.

$$\ln(1+x) = \int_0^x \frac{1}{1+t} dt = x - \frac{x^2}{2} + \frac{x^3}{3} + \ldots + (-1)^n \frac{x^{n+1}}{n+1} + \ldots (-1 < x < 1)$$
$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}$$

b) Use power series representaion of $\ln(1+x)$ to evaluate, $\lim_{x \to 0} \frac{x - \ln(1+x)}{x^2}$.

$$\lim_{x \to 0} \frac{x - \left(x - \frac{x^2}{2} + \frac{x^3}{3} + \ldots \right)}{x^2} = \lim_{x \to 0} \frac{\left(\frac{x^2}{2} - \frac{x^3}{3} + \ldots \right)}{x^2} = \frac{1}{2}$$

Q-6) (20 points) Let C be the curve represented by $r(t) = (\sin 2t)i + (\cos 2t)j + tk$, $0 \le t \le \pi$ .Then find
.

a) Arc length of C between $0 \le t \le \dfrac{\pi}{2}$.

$$\text{Arc Length}=\int_0^{\frac{\pi}{2}} \sqrt{(2\cos 2t)^2 + (-2\sin 2t)^2 + 1}\,dt = \int_0^{\frac{\pi}{2}} \sqrt{5}dt = \sqrt{5}t\Big|_0^{\frac{\pi}{2}} = \frac{\sqrt{5}\pi}{2}$$

b) Unit tangent vector to the curve C at $t = \dfrac{\pi}{4}$.

$$r'(t) = 2\cos 2t\vec{i} - 2\sin 2t\vec{j} + \vec{k}$$

$$\|r'(t)\| = \sqrt{4\cos^2 2t + 4\sin^2 2t + 1} = \sqrt{5}$$

$$\frac{r'(t)}{\|r'(t)\|} = \frac{1}{\sqrt{5}}\left(2\cos 2t\vec{i} - 2\sin 2t\vec{j} + \vec{k}\right)$$

$$\frac{r'(\frac{\pi}{4})}{\left\|r'(\frac{\pi}{4})\right\|} = \frac{1}{\sqrt{5}}\left(2\cos 2\frac{\pi}{4}\vec{i} - 2\sin 2\frac{\pi}{4}\vec{j} + \vec{k}\right) = \left\langle 0, \frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right\rangle$$