MATH 152 CALCULUS II

FINAL EXAMINATION SOLUTION

Eastern Mediterranean University

Department of Mathematics

Faculty of Arts and Sciences

FALL 2015-2016

Faculty of Arts and Sciences
Department of Mathematics
MATH152, Calculus II - Final Exam

Std.No: **Name –Surname** **12.01.2016**

1	2	3	4	5	6	TOTAL

Duration : 120 mins

SOME USEFUL FORMULAS

$$\oint_C F\cdot T\,ds = \oint_C \vec{F}\cdot d\vec{r} = \iint_S \left(curl\ \vec{F}\right)\cdot\vec{n}\,ds$$

$$\oint_C M(x,y)dx + N(x,y)dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)dA$$

$$\iint_S \vec{F}\cdot\vec{n}\,ds = \iiint_Q \vec{\nabla}\cdot\vec{F}\,dV$$

$$\vec{\nabla} = \frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k$$

$$\iint_S f(x,y,z)\,dS = \iint_R f(x,y,g(x,y))\sqrt{\left(g_x(x,y)\right)^2 + \left(g_y(x,y)\right)^2 + 1}\,dA$$

$$\int_C F\cdot dr = \int_C M(x,y,z)dx + N(x,y,z)dy + P(x,y,z)dz$$

Q-1) Given $F(x,y,z) = xy\,i + (y^2 + zx)\,\mathrm{j} + xyz\,k$. Then evaluate

a) (10 points) $\mathrm{div}F = \nabla\cdot F$

$$\vec{\nabla}.\vec{F} = \frac{\partial}{\partial x}(xy) + \frac{\partial}{\partial y}\left(y^2 + zx\right) + \frac{\partial}{\partial z}(xyz)$$

$$= y + 2y + xy = 3y + xy = y(3+x)$$

b) (10 points) $curl\ F = \nabla\times F$

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & y^2 + zx & xyz \end{vmatrix} = \vec{i}(xz - x) - \vec{j}(yz - 0) + \vec{k}(z - x)$$

$$= \langle xz - x, -yz, z - x\rangle$$

Q-2) a) (5 points) Show that the line integral $\int_C \left(3x^2y+5y^2\right)dx+\left(x^3+10xy+4\right)dy$ is independent of path.

$M(x,y,z)=3x^2y+5y^2,\quad N(x,y,z)=x^3+10xy+4$

$$\left.\begin{array}{l}\dfrac{\partial N}{\partial x}=3x^2+10y\\ \dfrac{\partial M}{\partial y}=3x^2+10y\end{array}\right\}\Rightarrow \frac{\partial N}{\partial x}=\frac{\partial M}{\partial y}\ \text{so integeral is independent of path}$$

b) (10 points) Find a potential function for $F(x,y,z)=\left(3x^2+5y^2\right)i+(x^3+10xy+4)$ j

If $f(x,y)$ is a potential function for $F(x,y)$ then

$f_x(x,y)=3x^2y+5y^2.....(1)$

$f_y(x,y)=x^3+10xy+4.....(2)$

From (1) $\Rightarrow f(x,y)=x^3y+5xy^2+h(y)\Rightarrow \dfrac{\partial f}{\partial y}=x^3+10xy+h'(y)$

From (2) $\Rightarrow \dfrac{\partial f}{\partial y}=x^3+10xy+h'(y)=x^3+10xy+4\Rightarrow h'(y)=4\Rightarrow h(y)=4y+c$

$\Rightarrow f(x,y)=x^3y+5xy^2+4y+c$

c) (5 points) Evaluate $\int_{(0,1)}^{(1,2)}\left(3x^2y+5y^2\right)dx+\left(x^3+10xy+4\right)dy$.

$=x^3y+5xy^2+4y\Big|_{(0,1)}^{(1,2)}=26$

Q-3) (10 points) Find a parametric equation for the curve C that is the part of the curve $y=x^2+2$ from $(0,2)$ to $(1,3)$ and evaluate the line integral $\int_C (3xy+x^2)dx+\left(2xy\right)dy$.

$x=t,\ y=t^2+2,\quad 0\le t\le 1$

$dx=dt, dy=2tdt$

$$\int_C\left(3xy+x^2\right)dx+\left(2xy\right)dy=\int_0^1\left(3t\left(t^2+2\right)+t^2+2t\left(t^2+2\right)2t\right)=\int_0^1\left(4t^4+3t^3+9t^2+6t\right)dt$$

$$=\frac{4}{5}t^5+\frac{3}{4}t^4+3t^3+3t^2\Big|_0^1=\frac{151}{10}$$

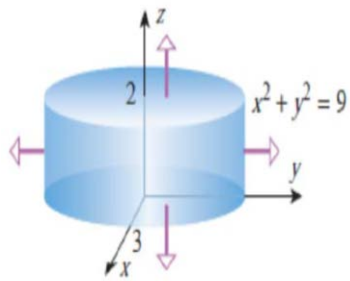
Q-4) (20 points) Use Green's Theorem to evaluate the line integral

$$\oint_C (x^3+xy)dx+(\cos y+x^2)dy$$ where C is the positively oriented boundary of the region,

bounded by $y=\sqrt{x}$, $x=4$ and $x-$ axis.

$$M(x,y)=x^3+xy,\ \ N(x,y)=\cos y+x^2$$
$$M_y=x,\ \ N_x=2x\Rightarrow$$
$$\int_C (x^3+xy)dx+(\cos y+x^2)dy=\iint_R (2x-x)\,dydx=\int_0^4\int_0^{\sqrt{x}} xdydx$$
$$\int_0^4 xy\Big|_0^{\sqrt{x}} dx=\int_0^4 x^{\frac{3}{2}}dx=\left.\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right|_0^4=\frac{2}{5}(4)^{\frac{5}{2}}=\frac{64}{5}$$

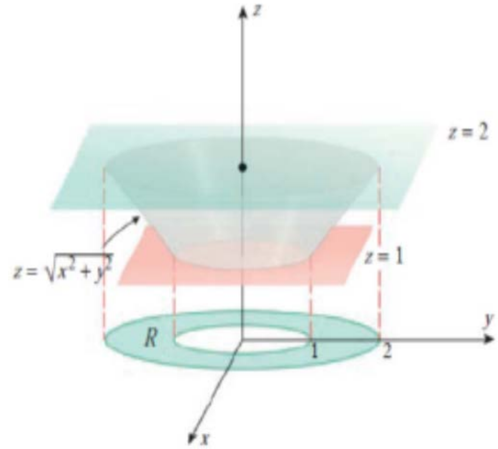
Q-5) (20 points) Use Divergence Theorem to evaluate the flux $\iint_S F\cdot\vec{n}\,dS$ of the vector field $F(x,y,z)=xy^2i+y\,j+zx^2k$ accross the surface of the region that is enclosed by the the cylinder $x^2+y^2=9$ and the planes $z=0$ and $z=2$.



Use cylindirical coordinates

$$x=r\cos\theta$$
$$y=r\sin\theta$$
$$z=z$$
$$dV=rdzdrd\theta$$
$$\vec{\nabla}.\vec{F}=y^2+1+x^2\Rightarrow\iint_S \vec{F}.\vec{n}dS=\iiint_Q (x^2+y^2+1)dV=\int_0^{2\pi}\int_0^3\int_0^2 (r^2+1)rdzdrd\theta$$
$$=2\int_0^{2\pi}\int_0^3 (r^3+r)drd\theta=2\int_0^{2\pi}\left(\frac{r^4}{4}+\frac{r^2}{2}\right)\Big|_0^3 d\theta=2\int_0^{2\pi}\left(\frac{3^4}{4}+\frac{3^2}{2}\right)d\theta=\frac{198}{4}2\pi=99\pi$$

Q-6) (20 points) Evaluate the surface integral $\iint_S z^2 dS$ where S is the part of the cone $z=\sqrt{x^2+y^2}$ that lies between the planes $z=1$ and $z=2$



$$\iint_S z^2 dS = \iint_{S_{xy}} (x^2+y^2)\sqrt{\frac{x^2}{x^2+y^2}+\frac{y^2}{x^2+y^2}+1}\,dA = \sqrt{2}\iint_{S_{xy}} (x^2+y^2)dA$$

$$= \sqrt{2}\int_0^{2\pi}\int_1^2 r^3 dr d\theta = \sqrt{2}\,2\pi \left.\frac{r^4}{4}\right|_1^2 = \sqrt{2}\pi\frac{15}{2}$$