CALCULUS II

FINAL EXAM SOLUTION

FALL 2016-2017

EMU
FACULTY OF ARTS AND SCIENCES
DEPARTMENT OF MATHEMATICS

Std.No:	**Name –Surname**					**03.01.2017**
1	**2**	**3**	**4**	**5**	**6**	**TOTAL**

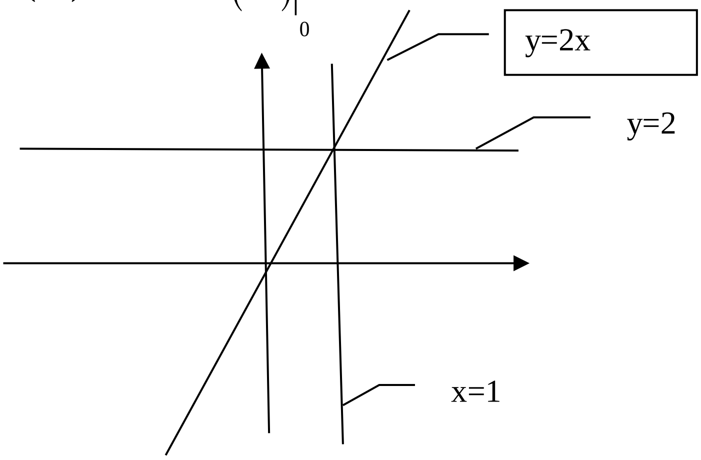
Duration : 120 minutes

Q-1) Reverse the order of the integration and evaluate $\int_0^2\int_{\frac{y}{2}}^1 \sin(x^2)dxdy$

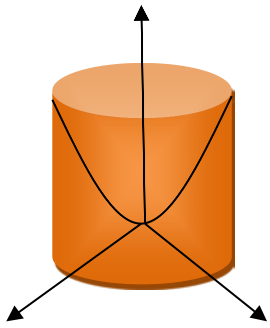
$$\left.\begin{array}{l} 0\le y\le 2 \\ \frac{y}{2}\le x\le 1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 0\le x\le 1 \\ 0\le y\le 2x \end{array}\right\}$$

$$\int_0^2\int_{\frac{y}{2}}^1 \sin(x^2)dxdy = \int_0^1\int_0^{2x} \sin(x^2)dydx = \int_0^1 y\sin(x^2)\Big|_0^{2x} dx =$$

$$\int_0^1 2x\sin(x^2)dx = -\cos\left(x^2\right)\Big|_0^1 = -\cos 1 + 1 = 1-\cos 1$$



Q-2) Use triple integral to find the volume of the solid region which lies between paraboloid $z = x^2 + y^2$ and the cylinder $x^2 + y^2 = 4, \ (z \geq 0)$.



$$Q = \begin{Bmatrix} 0 \leq \theta \leq 2\pi \\ 0 \leq r \leq 2 \\ 0 \leq z \leq r^2 \end{Bmatrix}$$

$$Vol = \iiint_Q 1dV = \int_0^{2\pi}\int_0^{2}\int_0^{r^2} rdzdrd\theta = \int_0^{2\pi}\int_0^{1} r^3drd\theta = \int_0^{2\pi} \left.\frac{r^4}{4}\right|_{r=0}^{r=2} d\theta$$

$$= 4(2\pi) = 8\pi$$

Q-3) (10 points) Evaluate the line integral $\oint_C \left(e^{x^2} - y^3\right)dx + \left(x^3 + e^{y^2}\right)dy$ where C is positively oriented boundary of the following region.

By Green's Formula

$$\oint_C \left(e^{x^2} - y^3\right)dx + \left(x^3 + e^{y^2}\right)dy$$

$$= \iint_Q 3\left(x^2 + y^2\right)dA = \int_0^{\pi}\int_1^{2} 3r^3drd\theta$$

$$\left.\frac{3r^4}{4}\right|_{r=1}^{r=2} \pi = \frac{3(16-1)}{4}\pi = \frac{45\pi}{4}$$

Q-4) (20 points) Evaluate the surface integral $\iint_S (2x+2y+z)\,dS$ where S is the first octant portion of the plane $x+y+z=2$.

$$z = 2-x-y \Rightarrow f_x = -1,\ f_y = -1$$

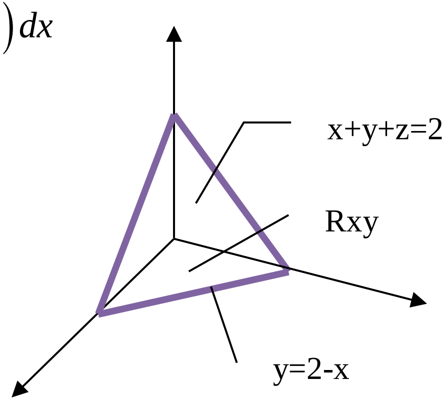
$$\iint_S (2x+y+z)dS = \iint_{R_{xy}} (2x+2y+2-x-y)\sqrt{(-1)^2+(-1)^2+1}dA$$

$$= \sqrt{3}\int_0^2\int_0^{2-x}(x+y+2)dydx = \sqrt{3}\int_0^2\left[(x+2)y+\frac{y^2}{2}\right]_0^{2-x} dx$$

$$= \sqrt{3}\int_0^2\left[(x+2)(2-x)+\frac{(2-x)^2}{2}\right]dx$$

$$= \sqrt{3}\int_0^2\left(\left[2x-x^2+4-2x\right]+\frac{4-4x+x^2}{2}\right)dx = \frac{\sqrt{3}}{2}\int_0^2\left(-x^2+12-4x\right)dx$$

$$= \frac{\sqrt{3}}{2}\left[\frac{-x^2}{3}+12x-2x^2\right]_0^2 = \frac{\sqrt{3}}{2}\frac{40}{3} = \frac{20}{\sqrt{3}}$$



Q-5) Given $F(x,y,z) = 3xzi + 4xy\,j + zxk$. Then evaluate

a) (6 points) $\text{div}F = \nabla\cdot F$

$$\text{div}F = \nabla\cdot F = 3z+4x+x = 3z+5x$$

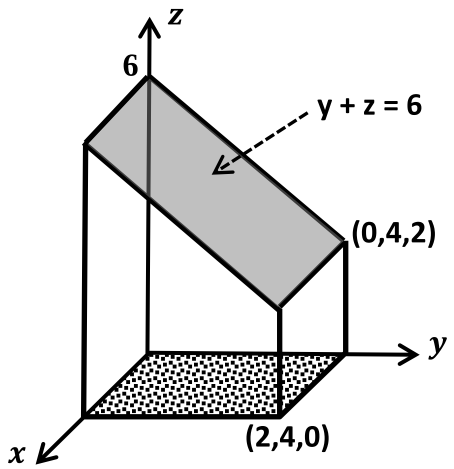
b) (8 points) $curl\ F = \nabla\times F$

$$curl\ F = \nabla\times F = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3xz & 4xy & 2x \end{vmatrix} = \vec{i}\,(0) - \vec{j}\,(2-3x) + \vec{k}\,(4y) = (3x-2)\,\vec{j} + 4y\vec{k}$$

c) (6 points) $\nabla\cdot(\nabla\times F)$

$$\nabla\cdot(\nabla\times F) = \frac{\partial}{\partial x}(0) + \frac{\partial}{\partial y}(3x-2) + \frac{\partial}{\partial z}(4y) = 0$$

Q-6) (20 points) Use Divergence Theorem to evaluate the flux $\iint_S F \cdot \vec{n}\, dS$ of the vector field $F(x,y,z) = yxi + y^2\, j + zk$ accross the surface of the region bounded by planes $x=0,\ x=2, y=0,\ y=4,\ z=0$ and $y+z=6$ (the region given below).



$$\iint_S F \cdot \vec{n}\; dS = \iiint_Q \nabla F dV \Rightarrow \text{Divergence theorem}$$

$$\nabla.F = y + 2y + 1 = 3y + 1$$

$$\iint_S F \cdot \vec{n}\; dS = \iiint_Q (3y+1)\, dV = \int_0^2 \int_0^4 \int_0^{6-y} (3y+1)\, dzdydx$$

$$\int_0^2 \int_0^4 (6-y)(3y+1)\, dydx = \int_0^2 \int_0^4 \left(17y + 6 - 3y^2\right) dydx$$

$$= \int_0^2 \left(-y^3 + \frac{17}{2} y^2 + 6y \Big|_0^4 \right) dx = \int_0^2 \left((-64 + 136 + 24) - 0 \right) dx = \int_0^2 96 dx = 96x\Big|_0^2 = 192$$