Eastern Mediterranean University

Calculus II

MT-1 Examination Solution

Department of Mathematics

2018-19 **FALL**

Faculty of Arts and Sciences
Department of Mathematics
MATH152 -Midterm I

Std.: **Name –Surname** **Gr:** **26.11.2018**

1	2	3	4	5	6	TOTAL

Duration : 90 mins

Q-1) (20 points) Find the center, radius and interval of convergence for $\sum_{n=1}^{\infty}\frac{(4x-3)^n}{5^{n+1}}$

$$\lim_{x\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{x\to\infty}\left|\frac{(4x-3)^{n+1}5^{n+1}}{5^{n+2}(4x-3)^n}\right| = \left|\frac{(4x-3)}{5}\right|.$$

It is convergent for, $\frac{|4x-3|}{5}<1 \Rightarrow -\frac{1}{2}<x<2$, and divergent for $x<-\frac{1}{2}$ and $x>2$.

If $x=-\frac{1}{2}$ then $\sum_{n=1}^{\infty}\frac{(4x-3)^n}{5^{n+1}} = \sum_{n=1}^{\infty}\frac{(-1)^n}{5}$ and divergent from alternating series test.

If $x=2$, then $\sum_{n=1}^{\infty}\frac{(4x-3)^n}{5^{n+1}} = \sum_{n=1}^{\infty}\frac{1}{5}$ and divergent from n^{th} term test.

Center of convergence $x=\frac{3}{4}$.

Radius of convergence $R=\frac{5}{4}$.

Interval of convergence $(-\frac{1}{2},2)$.

Q.2) (10 points) a) Use the following power series representation

$$\frac{1}{1+x}=1-x+x^2-x^3+\cdots+(-1)^n x^n+\cdots\ ,\ (-1<x<1)$$

to find the power series representations of $f(x)=(x+1)\ln(x+1)$ in powers of x.

$$\int\frac{1}{1+x}dx=\int(1-x+x^2-x^3+\cdots+(-1)^n x^n+\cdots)dx,\ -1<\mathrm{x}<1.$$

$$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots+(-1)^n\frac{x^{n+1}}{n+1}+\cdots,\ -1<\mathrm{x}<1.$$

$$(x+1)\ln(1+x)=x+\frac{1}{2}x^2-\frac{x^3}{6}+\frac{x^4}{12}+\cdots,\ -1<\mathrm{x}<1.$$

b) (5 points) Find the taylor polynomial $P_3(x)$, for the power series representation of $f(x)=(x+1)\ln(1+x)$ obtained in part (a).

$$P_3(x)=x+\frac{1}{2}x^2-\frac{x^3}{6}$$

c) (5 points) Use $P_3(x)$ obtained in b) to find the approximate value of $\ln(1/2)$.

$$P_3(x)=x+\frac{1}{2}x^2-\frac{x^3}{6}\approx(x+1)\ln(x+1)$$

$$(x+1)\ln(x+1)\approx x+\frac{1}{2}x^2-\frac{x^3}{6},\ \text{taking } x=-\frac{1}{2},$$

$$(1/2)\ln(1/2)\approx-\frac{1}{2}+\frac{1}{2}\left(\frac{1}{2}\right)^2-\frac{1}{6}\left(\frac{1}{2}\right)^3=\frac{1}{2}+\frac{1}{8}-\frac{1}{48}=\frac{-24+6-1}{48}=\frac{-19}{48}.$$

$$\ln(1/2)=\frac{-38}{48}=-\frac{17}{24}.$$

Q-3) (10 points) a) Find the intersection point of lines

$$l_1 : x = 3 + 2t, \;\; y = -2 + 2t, \;\; z = 1 - 4t, \quad -\infty < t < \infty$$

and

$$l_2 : x = -1 + v, \; y = -v, \;\; z = v, \quad -\infty < v < \infty \,.$$

$$\left.\begin{array}{l} -2t + v = 4 \cdots (1) \\ 2t + v = 2 \;\; \cdots (2) \\ 4t + v = 1 \;\; \cdots (3) \end{array}\right\} \Rightarrow \textit{from } (1) \textit{ and } (2) \;\; v = 3 \;\; \textit{and} \;\; t = -\frac{1}{2} \quad .$$

$$\textit{Use} \;\; v = 3 \; \textit{in} \;\; l_1 \;\; \textit{or} \; t = -\frac{1}{2} \;\; \textit{in} \;\; l_2 \Rightarrow P(2, -3, 3) \text{ is the intersection point}.$$

b) (10 points) Find the equation of the plane that contains l_1 and l_2 .

$$n_1 = \langle 2, 2, -4 \rangle, \quad n_2 = \langle 1, -1, 1 \rangle \Rightarrow n = n_1 \times n_2 = \begin{vmatrix} i & j & k \\ 2 & 2 & -4 \\ 1 & -1 & 1 \end{vmatrix} = -2i - 6j - 4k.$$

Equation of the plane:

$$-2(x-2) - 6(\mathrm{y}+3) - 4(z-3) = 0,$$

$$-2x - 6y - 4z = 2.$$

$$x + 3y + 2z = -1.$$

Q-4) (20 points) Given $\vec{u} = \langle -2,1,3 \rangle$ and $\vec{v} = \langle 1,-2,4 \rangle$ then find

a) (8 points) the angle between $2\vec{u}+\vec{v}$ and $\vec{u}-2\vec{v}$.

$$2\vec{u}+\vec{v} = 2\langle -2,1,3 \rangle + \langle 1,-2,4 \rangle = \langle -3,0,10 \rangle.$$

$$\vec{u}-2\vec{v} = \langle -2,1,3 \rangle - 2\langle 1,-2,4 \rangle = \langle -4,5,-5 \rangle.$$

$$\cos\theta = \frac{\langle -3,0,10 \rangle \cdot \langle -4,5,-5 \rangle}{\left\| \langle -3,0,10 \rangle \right\| \left\| \langle -4,5,-5 \rangle \right\|} = \frac{-38}{\sqrt{109}\sqrt{66}}$$

$$\theta = \cos^{-1}\left(\frac{-38}{\sqrt{109}\sqrt{66}} \right).$$

b) (5 points) the unit vector in the direction of $2\vec{u}+\vec{v}$.

$$2\vec{u}+\vec{v} = \langle -3,0,10 \rangle \Rightarrow \frac{2\vec{u}+\vec{v}}{\left\| 2\vec{u}+\vec{v} \right\|} = \left\langle \frac{-3}{\sqrt{109}}, 0, \frac{10}{\sqrt{109}} \right\rangle$$

c) (7 points) the vector projection of $\vec{u}$ onto $\vec{v}$.

$$proj_{\vec{v}}\, \vec{u} = \frac{\vec{u}\cdot\vec{v}}{\left\| \vec{v} \right\|}\vec{v} = \frac{\langle -2,1,3 \rangle \cdot \langle 1,-2,4 \rangle}{\left\| \langle 1,-2,4 \rangle \right\|^2} \langle 1,-2,4 \rangle = \frac{8}{21}\langle 1,-2,4 \rangle = \left\langle \frac{8}{21}, \frac{-16}{21}, \frac{32}{21} \right\rangle$$

Q-5) a) (10 points) Find all first order partial derivatives of $f(x,y,z)=x^2yz+(y^2+zx)^2$.

$$f(x,y,z)=x^2yz+(y^2+zx)^2$$

$$f_x(x,y,z)=2xyz+2(y^2+zx)z.$$
$$f_y(x,y,z)=x^2z+2(y^2+zx)2y.$$
$$f_z(x,y,z)=x^2y+2(y^2+zx)x.$$

b) (10 points) Use Chain rule to evaluate $\frac{\partial w}{\partial x}$ where $w=u^2v-v^2u$, $u=x^2y$ and $v=2x$.

$$\frac{\partial w}{\partial x}=\frac{\partial w}{\partial u}\frac{\partial u}{\partial x}+\frac{\partial w}{\partial v}\frac{dv}{dx}=(2uv-v^2)2xy+(u^2-2vu)2$$
$$=\left[2(x^2y)2x-4x^2\right]2xy+\left[(x^2y)^2-2(x^2y)2x\right]2$$
$$=\left[4x^3y-4x^2\right]2xy+\left[2x^4y^2-8x^3y\right]$$
$$=10x^4y^2-16x^3y.$$

Q-6) Evaluate the following limits.

a) (10 points) $\lim_{(x,y)\to(1,-2)} \frac{2x^2y}{x^4+2y^2}$.

$$\lim_{(x,y)\to(1,-2)} \frac{2x^2y}{x^4+2y^2} = \frac{2(-2)}{1+2(4)} = \frac{-4}{9}.$$

b) (10 points) $\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^4+2y^2}$.

$$y = mx^2 \Rightarrow \lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^4+2y^2} = \lim_{(x,y)\to(0,0)} \frac{2x^2(mx^2)}{x^4+2\left(mx^2\right)^2} = \lim_{(x,y)\to(0,0)} \frac{2mx^4}{x^4+2m^2x^4} = \frac{2m}{1+2m^2},$$

$$\left.\begin{array}{l} On\ \ y = x^2 \ (m=1), \text{ limit is } \frac{2}{3}, \\ On\ \ y = 2x^2 \ (m=2), \text{ limit is } \frac{4}{9}. \end{array}\right\} \Rightarrow \text{from two path rule limit does not exist.}$$